Experiment 2

Kinematics, Collisions and Simple Harmonic Motion

Introduction

In this experiment you will study motion with constant velocity, motion with constant acceleration, elastic and inelastice collisions and simple harmonic motion ($\mathcal{SHM}$). You will study one dimensional motion using an air-track system to reduce the effects of air resistance. To study $\mathcal{SHM}$, you will study the behaviour of a simple pendulum.

Ordinarily, it is difficult to examine motion with constant acceleration, since objects in free fall tend to move too rapidly, and frictional forces tend to arise in most everyday situations. These factors hinder a direct observation of the underlying physical principles of motion, and in fact this is one of the reasons why these principles were poorly understood until Galileo's famous experiments. In this lab, it will be possible to study motion in the absence of almost any friction by using a rider on a "frictionless" air-track. The air-track has rows of small air jets running down its side, which support the rider on a thin film of air and allow it to float just above the track. When the track is level and the rider is given a slight push, it will move with constant velocity; when the track is slightly inclined, the rider will experience a small acceleration due to the component of gravity which is parallel to the track.

To study the motion of the rider, you will also need to be able to make accurate measurements of its position at given intervals of time. For this you will use a sonar device called the Sonic Ranger. This apparatus sends out discrete pulses of sound waves, which are reflected back by the object or objects under observation. The sensor is the essentially the same as used in some commercially available autofocus cameras. The Sonic Ranger is in turn connected to a computer, which calculates the distance to the object based on the time it takes the signal to leave the sonar module and return. If a series of such measurements is made in rapid succession, then the computer can reconstruct the motion of the rider over some time interval, and this information can be used for many other calculations, such as the "instantaneous" velocity or acceleration of the rider as a function of time. An essential part of this lab is becoming familiar with using the computer to acquire and analyze data. Use of the computer offers many advantages over manipulating data by hand, but you can benefit from it only if you have learned how to use the computer effectively and understand its limitations.

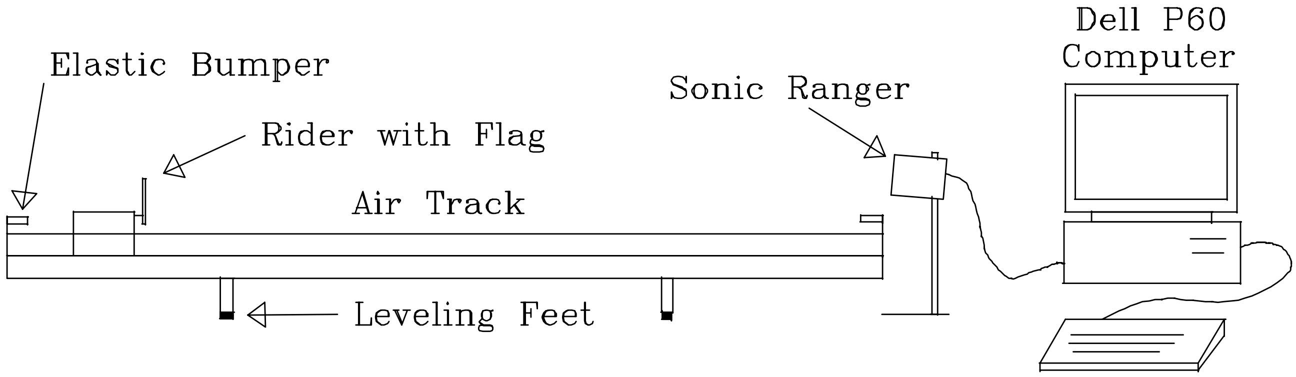


Figure 2.1 Schematic view of Sonic Ranger with air track.

2.1 Part I — Setup

2.1.1 Getting Started

To begin with, eiher power up reboot the computer by hitting `Ctrl-Alt-Delete` (all at the same time). If the computer is running Windows, you can exit by typing `Alt-F4`. Then at the `C:\ >` prompt type the boldface commands below:

`C:\ >`**cd\sonic**

`C:\SONIC>`**sr**

this will load the Sonic Ranger software. Make sure that the Sonic Ranger module is plugged in, and then begin by trying this simple experiment. Prepare the software to take data by selecting `Collect Data` from the `Main Menu`. Next choose `One Target`, and then hit the space-bar to skip past the screen with a beep. An empty graph should now appear, and the computer is ready to take data. Stand about six or eight feet away from the Sonic Ranger and have your lab partner point it toward you. When your partner presses a key to begin taking data, start moving slowly toward the Sonic Ranger. Do you think that you can move with a constant velocity? Try and see if you are able to. Are you able to tell from the graph of your position *vs.* time whether or not you have been reasonably successful? You may have to try a few times: to re-take data, simply hit `Return` after you are done and follow the instructions.

From this data, the software is able to construct a graph of velocity *vs.* time by calculating the average velocity between each of the measurements (the change in distance divided by the time interval). Since the measurements are taken very close together, the calculated velocity for each interval is reasonably close to the instantaneous velocity at any instant within that interval, as long as the rate of change of the velocity is small. To see a graph of velocity as a function of time, you must return to the `Main Menu`. Hit `Return` when you have finished taking data (hitting `escape` will erase your data, so make sure you hit `Return`) and press '`N`' to skip re-taking data. From the `Main Menu`, select `Analyse Data` and after that select `View Graphs`. This presents you with a menu of various options and settings. You can switch from one option to another by pressing the up- and down-arrow keys; you can change the setting of a given option by pressing the left- and right-arrow keys. For now, simply change the `Number of Graphs` from 1 to 2 by pressing the right-arrow key. Press '`V`' to view the graphs and hit the space-bar again to pass the beeping screen. You should now be presented with both a displacement graph and a velocity graph. How constant was your velocity?

Collect some more data (press `Return` to leave the graphs and go back to the `Main Menu`) and try making a more complicated motion. From the displacement graph, draw a prediction of the velocity graph and then go back to `Analyse Data` to see whether you are, roughly, correct.

2.1.2 Setting Up the Sonic Ranger

Set the Sonic Ranger just at the right end of the track. If the module is pointed straight horizontally, then the emitted sound waves will reflect off the track and back to the module before they reflect off the rider (See Fig. 2.2). Therefore, tilt the module slightly upward, about ten degrees, so that the sound waves will neither hit the track nor miss the rider when it is at the other end.

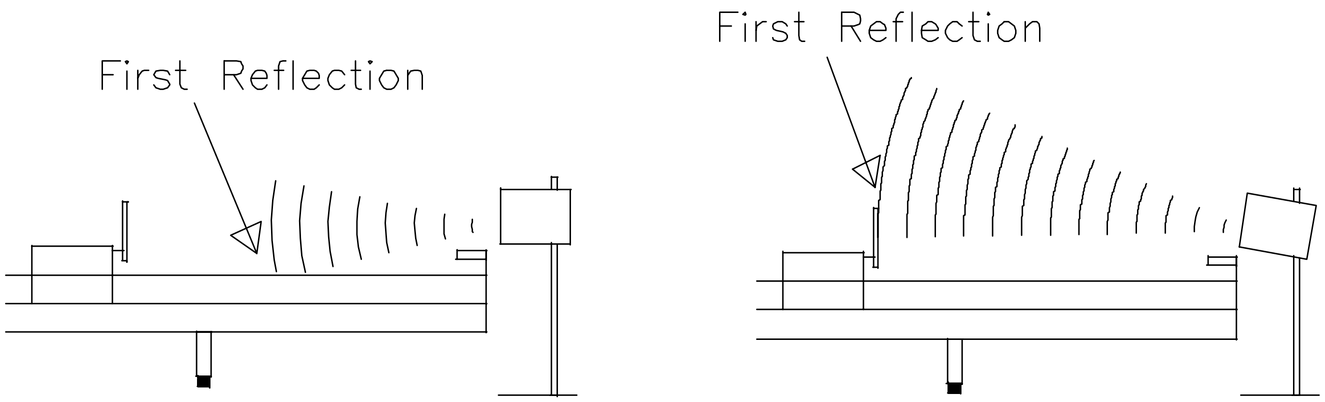


Figure 2.2 When the Sonic Ranger is aimed straight, the sound waves reflect first off the track, causing the apparatus to measure the distance to the track, not the rider. Tilt the Sonic Ranger slightly upward to avoid this.

Turn on the air to the air-track and set the rider at the far left end of the track. Prop up the right side of the track so that the rider stays at the left end–this will allow you to make sure the Sonic Ranger is able to detect the rider when it is at the far end. Follow the same procedure as you did before to collect data. The distance from the Sonic Ranger to the flag on the rider will be approximately 1.8 m, so if the module is aimed correctly you should see on the graph an object sitting 1.8 m away; if not, try to re-aim the module while you are taking data or try taking data again. You can then check that the alignment is good by giving the rider a small push along the air-track, and observe whether the computer plots the rider's position smoothly throughout the length of the track. If there are static-like jumps in the plot, then the sound waves are not reflecting properly back to the Sonic Ranger, and you will need to align it more carefully.

2.1.3 A Note about Close Distances

The Sonic Ranger is unable to detect objects closer than 41 cm, because it requires a certain amount of time between sending and receiving signals. Objects within this range will reflect signals too early for the Ranger to interpret the data correctly. **Therefore, always work with distances greater than 50 cm from the Sonic Ranger to insure that the distance to the object is determined correctly.**

Note also that the scale on the air-track increases as you read toward the right. Since the Sonic Ranger is aimed left, its scale increases toward the *left*, with the origin at the faceplate of the module, not the right end of the track (see Fig. 2.3). It is extremely important that you keep these two scales separate in your work. You should use the readings from the Sonic Ranger (which the computer indicates in *meters*) for all of your experimental calculations, and use the scale on the air-track (which is marked in *centimeters*) only for positioning the rider in the same place when you are repeating an experiment over several trials.

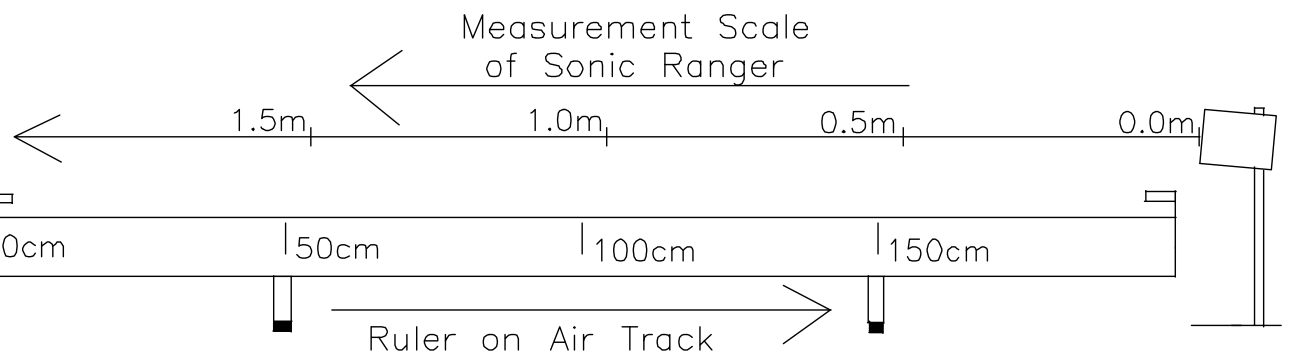


Figure 2.3 The measurement scale of the Sonic Ranger starts at its faceplate and increases toward the left, oppositely of the ruler on the air track.

2.1.4 Calibrating the Sonic Ranger Software

The Sonic Ranger software assumes the speed of sound in air is 340.0 m/s as a default value when you start the program. The actual value fluctuates slightly due to changes in temperature, and you should calibrate your equipment more precisely before you begin to make measurements.

From the `Main Menu`, select `Calibrate`, and within the submenu select `Calibrate By Measurement`. Follow the instructions on the screen: first set the rider on the air-track at some precise distance from the Sonic Ranger (at least 41 cm) and take data at this distance, then set the rider at a position exactly one meter further away and take data again. The computer will then calculate the speed of sound by using the extra time it took the sound to travel one extra meter and return. How much does this value differ from the default value of 340.0 m/s, and how much will that difference affect your results?

Press `Return` to accept this calibration.

2.1.5 Note About the Rider and Air-Track

It is important that you take care in using the rider and air-track. Don't let the rider sit on the track when the air is off, and don't let the flag-side of the rider collide with the elastic bumper, since both of these can cause the rider and air-track to scrape against each other, leaving scratches which permanently damage the equipment.

2.2 Part II — Motion with Constant Velocity

2.2.1 Leveling the Air Track

To study motion with constant velocity it will be necessary to level the air-track as carefully as possible so that the rider does not tend to accelerate in one direction. The left side of the air-track has two adjustable feet; the right foot is not adjustable, but you may raise it by stacking sheets of paper beneath it. (Avoid using the shims for leveling the track, since you will be using them later to raise it to a fixed inclination.) The air-track has been machined to remain extremely straight along its entire length, but because the rider is also extremely sensitive to the slightest variation along the track, you will find that the rider sometimes remains stationary in one region of the track but tends to drift when it is in another region. It is not always possible to completely level the track, but you should try to minimize these irregularities by making sure that the drift is as small as possible in most parts of the track and that the direction of the drifts are more or less random. (If all the drifts tend in the same direction, then this indicates that you can make the track more level.)

2.2.2 Taking Data

Once you are satisfied that the track is sufficiently level, set the rider at the 150 cm mark. Begin collecting data, and then give the rider a gentle push toward the left. Make sure that you are able to take data over a substantial portion of the return trip after it has bounced off the elastic bumper at the left end; also make sure that the data is smooth and without jumps. When you have collected this data, return to the `Main Menu` and proceed in the same manner as before to display a graph of displacement and a graph of velocity with respect to time.

Is the velocity graph what you expected? The time scales of the two graphs are the same, so you should be able to see how the rate of change in the displacement graph corresponds to the velocity. How does motion with constant velocity on the air-track compare with trying to walk with constant velocity?

2.2.3 The Coefficient of Restitution

When two objects collide and bounce away from each other, they tend to lose some of their energy in the collision, and the rebound velocity between the two objects is therefore less than the initial velocity between them. This is why objects that are dropped will sooner or later stop bouncing. The *elasticity* of the collision can be indicated by e, the **coefficient of restitution**, which is defined as the relative speed after the collision divided by the relative speed before the collision:

$$e = \left| \frac{v_{2f} - v_{1f}}{v_{2i} - v_{1i}} \right|$$

A perfectly elastic collision would therefore have a coefficient of restitution equal to one; an elastic "super ball" is a good example of an object whose coefficient of restitution in many collisions is often close to one.

2.2.4 Using the Cursor to Read Off Data Points

You can calculate e for the case of the rider colliding with the elastic bumper by using the data you collected in this experiment. There are two menus for operating on the data shown on the screen: they are displayed underneath the graphs, and you can switch back and forth between them by pressing '`N`' for `Next Menu` and '`P`' for `Previous Menu`. Find the menu with the `(C)ursor` option and hit '`C`'. This draws a vertical line on the graphs which acts like a cursor: you can move it left and right with the arrow keys, and you can move more quickly if you hold down the control key at the same time. At the bottom of the screen, the x value (time) of the cursor and the two corresponding y values (displacement in the top graph, velocity in the bottom) are indicated more precisely than they can be read off the graphs. By moving the cursor to times a little bit before and after the collision with the elastic bumper (regions where the velocity is still constant), you can read off the values of the velocity at those times, and then calculate e.

2.2.5 Constructing a Best Fit Line

Another method consists in using the software to calculate velocity based on a kind of average of a segment of the data, called a **best fit line**. To do this, you will first need to instruct the software which segment of data it should fit a line to. The software will fit a line to the entire segment of data shown on the graph, so to fit a line to just one part, you must rescale the graphs. Find the menu with the `(R)escale` option and hit '`R`' (you will have to exit the Cursor function, if you have not already done so, by pressing `Return`). A small cursor will appear in the upper-left corner of the upper graph. If you hit the space bar, it will move to the lower-right corner of the graph, and if you keep hitting the space bar it will move to these two corners of the second graph and then back to the top graph again. At each of these corners, you can use the arrow keys to adjust the scale of the graph: the up- and down-arrow keys will magnify or demagnify the vertical scale and the left- and right-arrow keys will magnify or demagnify the time scale. (Again, holding down the Control key will move things faster.) Note that the time scale will change on both graphs simultaneously, while the vertical scale changes separately for each graph. For now, you only need to be concerned with the horizontal rescaling — that is, with changing the first and last time values on the graph so that you are looking at just a part of the data. By playing around with rescaling in the left and right corners of the graph, using just the left- and right-arrow keys, you can eventually arrange it so that the graphs show *only* that segment of time during which the rider is in constant motion toward the elastic bumper. It is not necessary here to try to include every single data-point from the beginning until the point of collision; rather, you should try to focus on some window of points reasonably inside this range, so that extraneous effects at the ends of this motion will not show up in the graph (see Fig. 2.4).

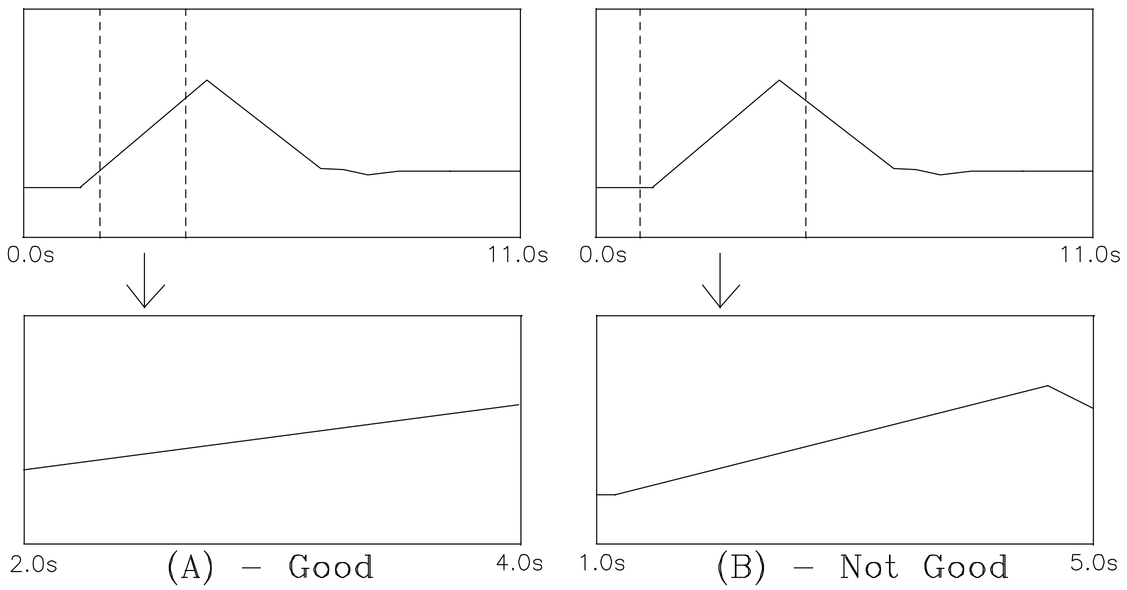


Figure 2.4 Example (A) shows how to rescale the graph to include just the segment of the data needed to get a best-fit line. In Example (B), too much of the data is included, and the best-fit line will not correspond to the desired segment of data.

When this is done, exit the Rescaling function by pressing `Return`. Find the menu with `(B)est Fit Line` and press '`B`'. The software draws a best fit line through the data points on each graph. It is something like the graphical equivalent of what happens when you take the average of several measurements. Using the information derived from a best-fit line should be more accurate than using individual data points, in the same way that using the average of several measurements of a quantity is more reliably accurate than using any single measurement.

At the bottom of the screen, the slopes of each of the best-fit lines are indicated by "M = ..." The slope of the displacement graph indicates a velocity, in this case the velocity of the rider moving toward the elastic bumper. The slope on the velocity graph indicates the acceleration. Are these what you expected them to be? How close is the velocity to what you measured for the velocity above?

Make a note of the velocity indicated by the best fit of the displacement graph and then rescale your windows so that only the smooth motion *away* from the elastic bumper and *toward* the Sonic Ranger is visible. Using the best-fit method again, find the velocity during this interval, and calculate the coefficient of restitution again. How would you explain any differences between the result of this calculation and the result of the method you used earlier? Which method do you think gives a more accurate value for e?

Using one of these methods, conduct several more trials and find an average value for e.

2.3 Part III — Gravitational Acceleration

A small constant force can be applied to the rider by inclining the track slightly. The component of gravity which acts on the rider parallel to the air-track is equal to a_x, as indicated in Fig. 2.5.

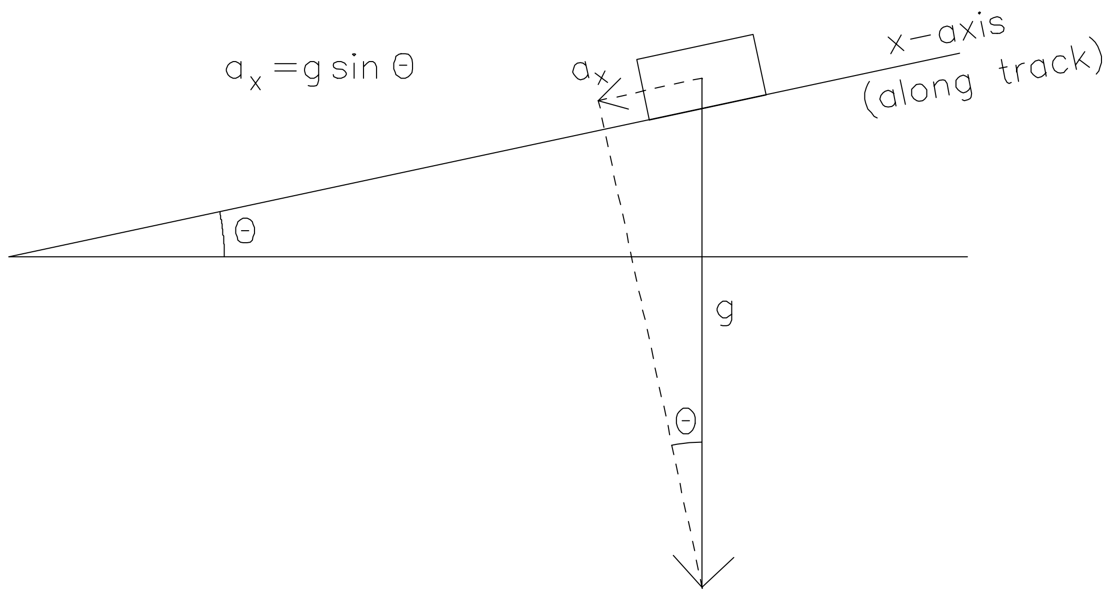
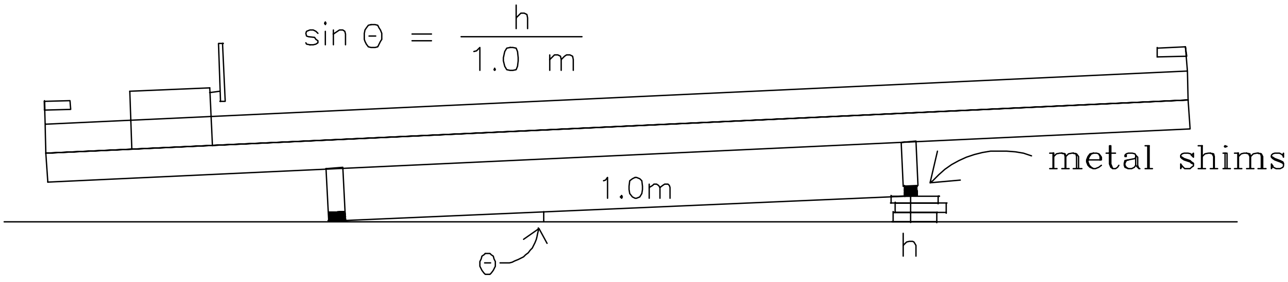


Figure 2.5 Geometry for rider on air-track with a small angle.

Use a few of the shims provided to elevate the right side of the air-track.

NOTE: All of the shims are 1.24 mm thick, <u>not</u> 1.32 mm as marked.

Note that $\sin\theta$ equals the height of the shims divided by the length of the track between the supports, which in this experiment is exactly 1.0 m (see Fig. 2.3).



A convenient method for taking data is as follows: proceed through the menus until the point when the empty graph is on the screen and the computer is ready to begin collecting data at the press of a key. At this point, set the rider on the track at a given point, say at the 100 cm mark, and release it. After the rider has bounced off the elastic bumper, begin taking data. You should be able to record the rest of the rider's trip up the track, and all of it's trip back down to the bumper again; if not, try taking this data again and waiting a little longer after the rider bounces off the end.

From this one series of measurements of the rider's position, there are many ways to analyze the data and calculate the acceleration of the rider. All of these ways are mathematically related, but for each one you are isolating and analyzing different pieces of the data, so the correspondence between different calculations will not be exact. There are four methods which you should consider:

$$
\begin{aligned}
x_2 &= x_1 + v_1\Delta t + \frac{1}{2}\Delta t^2 && (1)\\
v_2 &= v_1^2 + 2a_x(x_2 - x_1) && (2)\\
v_2 - v_1 &= a_x\Delta t && (3)\\
&\text{a best fit to } v && (4)
\end{aligned}
$$

Analyzing the data you just collected, calculate the value of a_x, and in turn g, using different data points from the graph for each of the equations above. Is your calculation of g close to the accepted value?

Perform the same analysis on two more trials. For the first, use a different inclination of the air-track. For the second, start with the rider at the bottom of the track (using either inclination) and after you begin taking data, give it a gentle push up the track.

Can you observe any significant deviation in the results obtained from these different trials? Which of the four methods seems most accurate? Using all of your results, what is your average calculated value of g?

2.4 Simple Harmonic Motion — Pendulum

In this part of the experiment you will study simple harmonic motion ($\mathcal{SHM}$) using a simple pendulum. Examples of $\mathcal{SHM}$ are found in many systems: oscillations of any system around a stable equilibrium point can usually be well-approximated in terms of $\mathcal{SHM}$.

Consider a simple pendulum consisting of a mass m suspended by a string of length l as shown below. Assume that the string is at an angle θ with respect to the vertical.

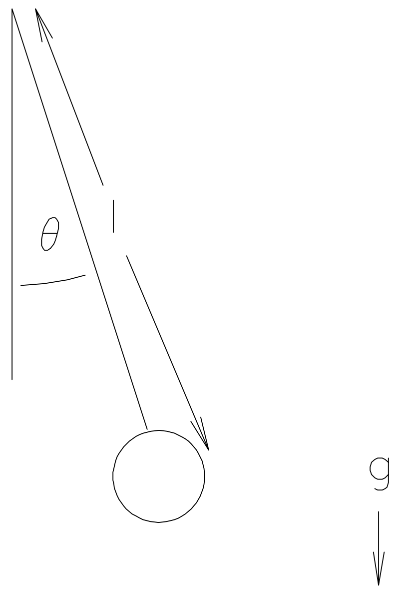


Figure 2.6 Simple Pendulum

The net torque acting on the mass, taking the point where the string is suspended as our origin, is given by

$$\tau = -mgl\sin\theta.$$

The moment of inertia I of the system is

$$I = ml^2.$$

The instantaneous angular acceleration is therefore given by

$$\begin{aligned} \tau &= I\,\frac{d^2\theta}{dt^2} \\ -mgl\sin\theta &= I\,\frac{d^2\theta}{dt^2} \end{aligned}$$

If the angle θ is small (say less than $15^\circ = 15 * 2\pi/360$ radians), then $\sin\theta \approx \theta$, where θ is measured in radians. We can then rearrange the above differential equation for θ to obtain:

$$\frac{d^2\theta}{dt^2} = -\frac{g}{l}\theta.$$

This is the equation for a simple harmonic oscillator ($\mathcal{SHO}$).

We can solve this equation by guessing the form of the solution, then substituting back into the equation. Guided by our knowledge of a mass on a spring (another system which obeys the $\mathcal{SHO}$ equation), we anticipate a solution where the angle θ is a sinusoidal function of time. The most general such solution is given by

$$\theta(t) = \theta_0 \cos(\omega t + \phi).$$

We can differentiate this equation twice to obtain

$$\frac{d^2\theta}{dt^2} = -\omega^2 \theta_0 \cos(\omega t + \phi) = -\omega^2 \theta(t),$$

then substitute the result into the $\mathcal{SHO}$ equation. Doing this, we find that

$$\omega^2 = \frac{g}{l}.$$

The values of θ_0 and ϕ depend on the initial conditions, they correspond to the amplitude of the oscillation (the maximum angle reached) and the phase of the oscillations (essentially depending on when we choose for the time $t = 0$ relative to the motion of the pendulum).

2.4.1 Measuring the Oscillation Frequency

Place the Sonic Ranger roughly 1 m from the stationary pendulum bob, then start the pendulum moving away and towards the Sonic Ranger. Using the Ranger software, measure the bob's horizontal displacement $x(t)$. For small amplitude motion, the displacement $x \approx l\theta$, so that the displacement is given by

$$x(t) = l\theta_0 \cos\left(\sqrt{\frac{g}{l}}t + \phi\right).$$

Once you have a good measurement of $x(t)$ without any loss of signal *etc.*, calculate the corresponding $v_x(t)$ and $a_x(t)$. Draw a graph of $x(t)$, $v_x(t)$, and $a_x(t)$, using a common time axis. Briefly discuss the relative phases of the three graphs.

Calculate the period and angular frequency of the pendulum's motion. Recall that $T = 2\pi/\omega$. Think about how to make the most accurate measurement of these quantities from your data, and describe your procedure. Measure the frequency of the pendulum motion for several different amplitudes of the oscillation.

Measure the length l of the pendulum, from the pivot point to the center of the bob. Using the relationship

$$\omega = \sqrt{\frac{g}{l}}$$

calculate the acceleration due to gravity g. Repeat this for a different value of the string length l. Compare your results for g with that obtained in the airtrack measurement and discuss which technique you expect to be more precise.