

Superconducting fluctuation corrections to ultrasound attenuation and sound velocity in layered superconductors.

CERPEMA

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The outline

- The fluctuations
- Sound attenuation in impure metals
- The fluctuation diagrams
- Calculations
- Experimental situation
- Results
- Conclusion

I. Fluctuation phenomena in layered superconductors

The fluctuation regime:

The Ginzburg number: the thermal energy vs. the condensation energy per coherence volume.

$$\text{in 3D materials: } G = \frac{|T - T_c|}{T_c} \sim \left(\frac{T_c}{\varepsilon_F}\right)^4 \sim 10^{-11} \div 10^{-14}$$

• in layered, quasi-2D materials: G can be up to $10^{-2} - 10^{-3}$

The demonstration of the fluctuation contribution to:

- conductivity
- thermoconductivity
- sound attenuation
- magnetoconductivity
- tunneling measurements
- Nernst effect

II. Previous theoretical works on sound attenuation

Sound attenuation in normal and superconducting metal with the impurities:

- A. B. Pippard, Philosophical magazine 46 (1955) 1104

Boltzmann equation formalism.

$$\alpha = \frac{Nm}{\rho v_s \tau} \left(\frac{1}{3} \frac{k^2 l^2 \tan^{-1} kl}{kl - \tan^{-1} kl} - 1 \right)$$

- T. Tsuneto, PR 121 (1961) 402: Normal metal and superconductor; BCS, density matrix, impurities.
- G. Kotliar, T.V. Ramakrishnan, PRB 31 (1985) 8188: Electron-phonon interaction in strongly disordered metals and sound attenuation, RPA resummation of the Coulomb interaction.
- M.Yu. Reizer, PRB 40 (1989) 7461: Attenuation in impure metals, various types of scattering (el-ph, el-magnon, el-impurity, weak localization).
- L.G. Aslamazov, A.A. Varlamov, JETP 77 (1979) 2410: Fluctuation effects in dirty superconductors and sound attenuation. Fröhlich model. Impurity corrections of electron-phonon vertices.

III. Model

- Longitudinal sound propagation perpendicular to the layers above the critical temperature
- The quasiparticle energy spectrum:

$$\xi(p_{\parallel}, p_{\perp}) = \frac{p_{\parallel}^2 - p_{\perp}^2}{2m} + t \cos p_{\perp} d.$$

(DOS does not depend on quasiparticle momentum)

- Moving reference frame
- Electron-phonon interaction in a tight-binding model

M.B. Walker, M.F. Smith, K.V. Samokhin, Phys. Rev. B 65 (2002) 014517

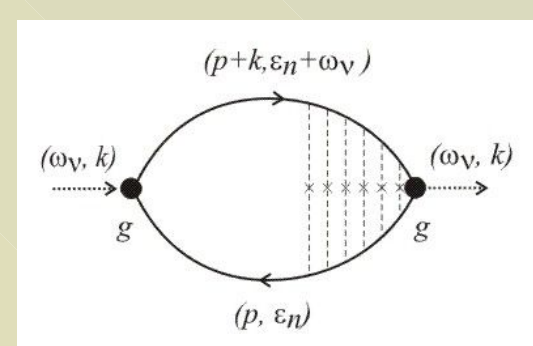
$$H_{e-ph} = -\sqrt{2}i \left(\frac{\hbar \omega_0(\mathbf{k})}{NMv_s^2} \right)^{1/2} G \sum_{\mathbf{k}, \mathbf{p}} (\cos p_x d) c_{\mathbf{p}+\mathbf{k}, \sigma}^{\dagger} c_{\mathbf{p}, \sigma} (a_{-\mathbf{k}}^{\dagger} + a_{\mathbf{k}})$$

- The attenuation is obtained from the imaginary part of the polarization operator $\Pi(\mathbf{k}, \omega)$ of the phonon Green's function $D(\mathbf{k}, \omega)$

$$D^{-1}(\mathbf{k}, \omega) = \left(D^0(\mathbf{k}, \omega) \right)^{-1} - \Pi(\mathbf{k}, \omega)$$

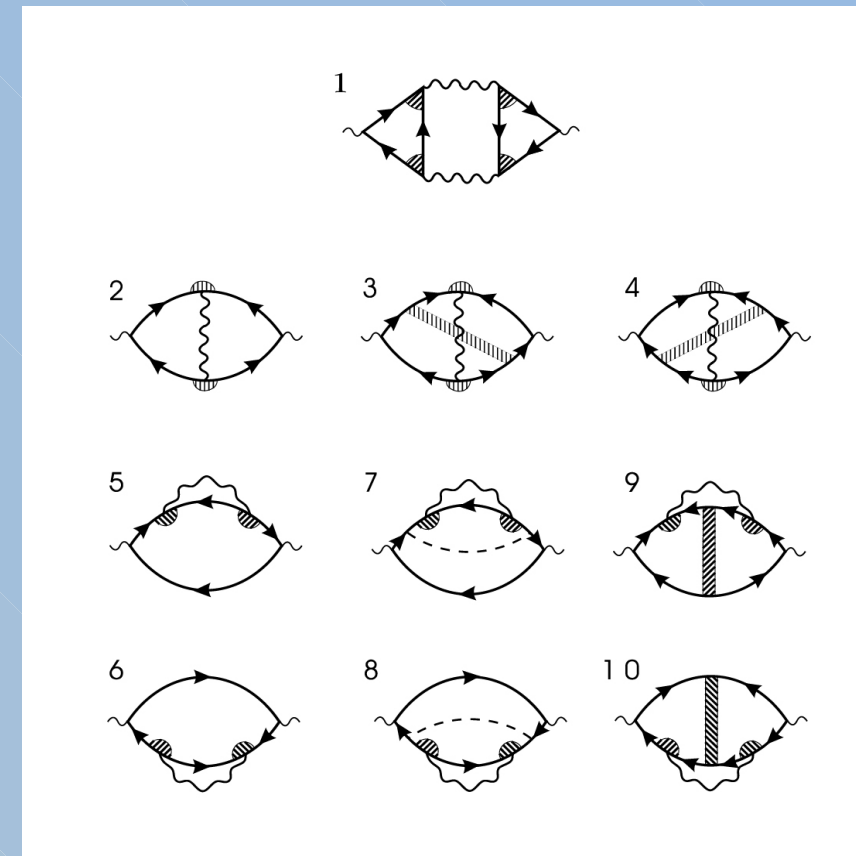
$$D^0(\mathbf{k}, \omega) = -\frac{\omega_0^2(\mathbf{k})}{\omega^2 + \omega_0^2(\mathbf{k})}$$

- Phonon self-energy with the impurity renormalization of electron-phonon vertices.



- The electron-phonon vertex g is then proportional to $\cos(p_x d)$

IV. Diagrams for the fluctuation sound attenuation



Feynman diagrams giving the leading-order contribution of the Aslamazov-Larkin (AL) (1), Maki-Thompson (MT) (2-4) and the density of states (DOS) (5-10) type.

A.I. Larkin, A.A. Varlamov, cond-mat/0109177 and *Handbook on Superconductivity: Conventional and Unconventional Superconductors*, edited by K.-H. Bennemann and J. Ketterson, (Springer)

G. Kotliar, T.V. Ramakrishnan Phys. Rev. B 31 (1985) 8188; A. Schmid Z. Physik 259 (1973) 42:

- For small k and ω : no diffusion enhancement of the electron-phonon vertex and of the attenuation.
- A consequence of the relaxation in a moving frame and of the screening.
- The fluctuation diagrams are similar to those for the conductivity.

V. Details

Difficulties

- Impurity vertex corrections.
- The diagrams with the impurity ladders in Cooper channel (3, 4, 9, 10) drop out for sound propagating in z-direction in layered material.
- Analytic continuation of anomalous MT diagram.
- «Regular anomalous» MT term.

Details and results

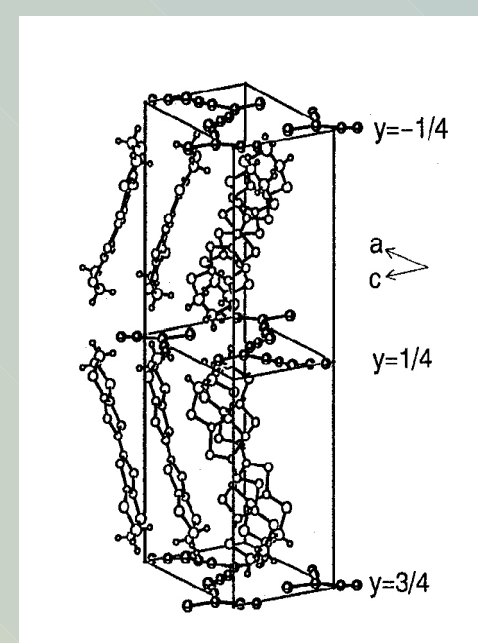
The expansion at $\omega \rightarrow 0$, $k \rightarrow 0$ in leading order gives: $k^0(\omega^0 + i\omega^1 + \omega^2)$

	ω^0	ω^1	ω^2
DOS	+	-	+
rMT	-	+	-
aMT	0	-	+

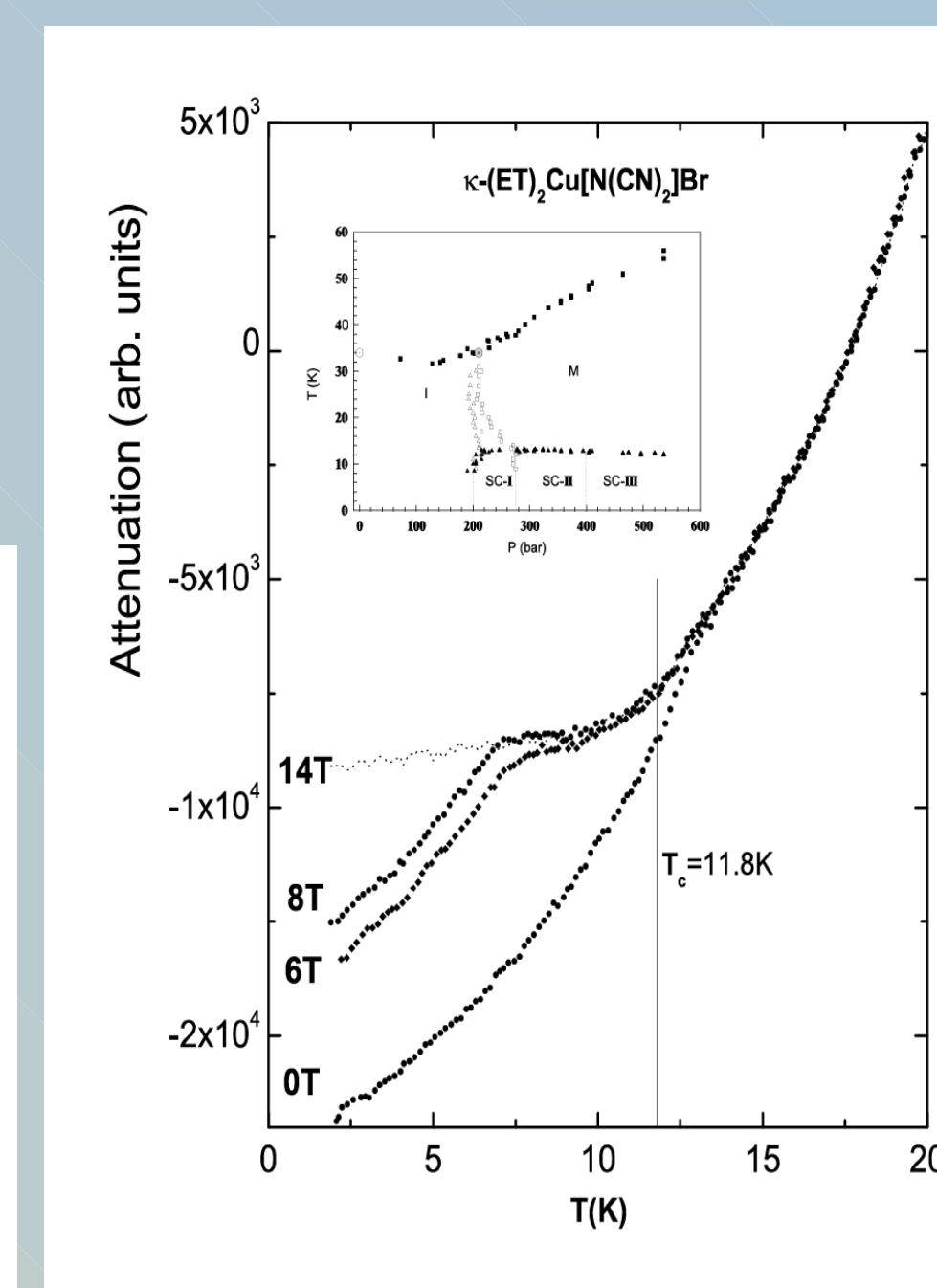
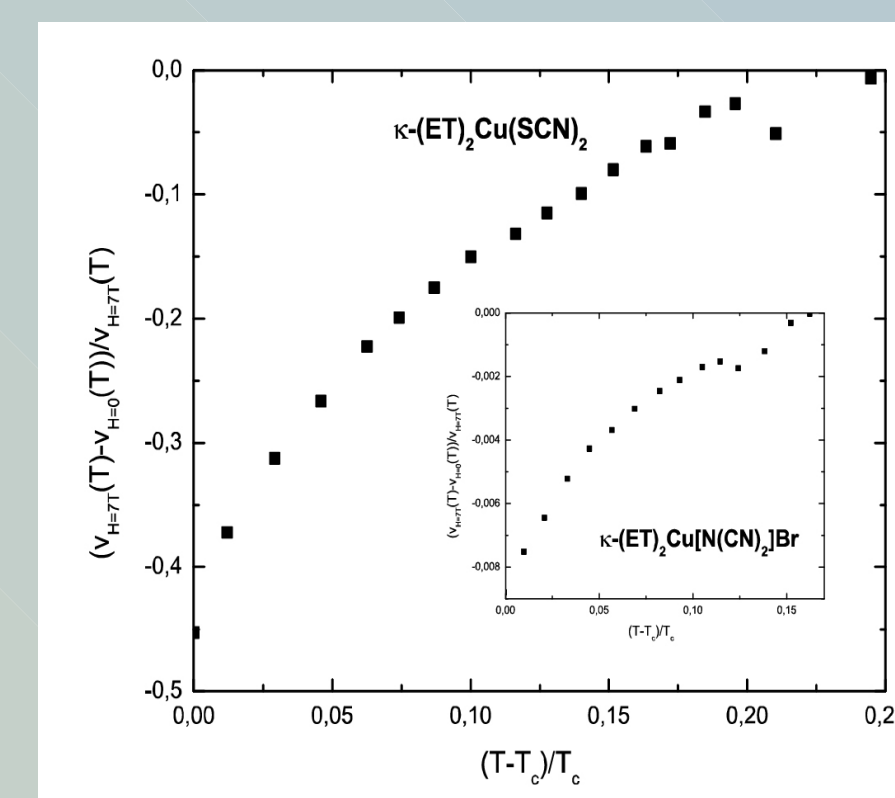
- Two types of MT term: the regular MT and the anomalous MT.
- Each of DOS, rMT and aMT types has its own temperature dependence.
- There are zero-order terms in rMT and DOS diagrams, while the anomalous MT term drops out in ω^0 order.
- The actual balance of DOS and MT contributions depends on the material parameters: the anisotropy parameter and the phase-breaking time.
- The AL diagram integrates to zero at given energy spectrum.
- aMT diagram seems to be the most important one for the attenuation, while the DOS gives the leading order in the sound velocity.

VI. Ultrasound measurements in the University of Sherbrooke

K. Frikach, M. Poirier, M. Castonguay, and K. D. Truong Phys. Rev. B 61, R6491-R6494 (2000)
D. Fournier, M. Poirier, M. Castonguay, K. Truong, cond-mat/0209536



The crystal structure of κ -(BEDTTTF)₂Cu[N(CN)₂]Br. The conducting plane is a-c one. $T_c = 11.8\text{K}$ (at 300 bar).



Sound velocity and sound attenuation data in magnetic field.

VII. Results

Attenuation

The imaginary part of the phonon frequency

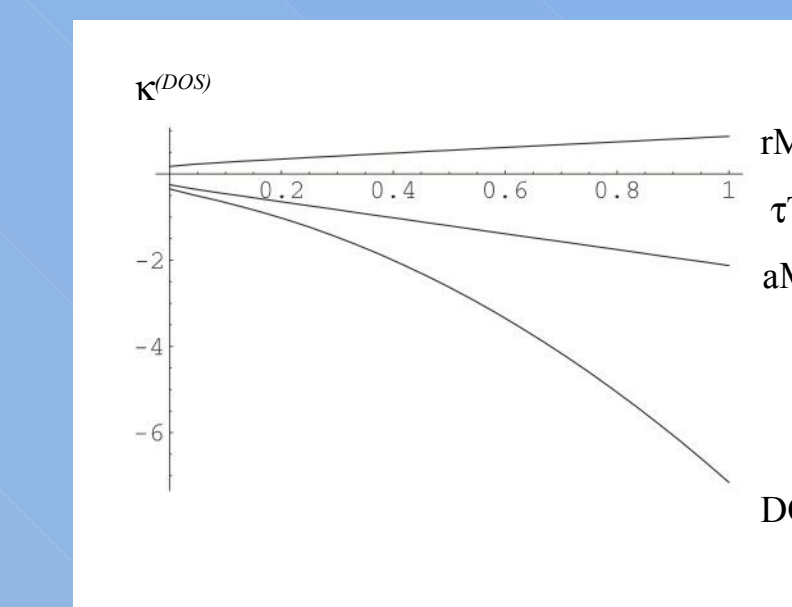
$$\delta(\omega) = \frac{1}{2} \omega(k) \text{Im} \Pi(\omega)$$

The power attenuation is then:

$$\alpha = -2\delta(\omega)/v_s$$

$$\alpha^{\beta} = \frac{g^2 \omega^2}{d v_s^2 v_n} \kappa_{att}^{\beta}(T, \tau) f^{\beta}(\epsilon, r, \gamma_{\phi})$$

The coefficients:



Sound velocity

The phonon frequency renormalization

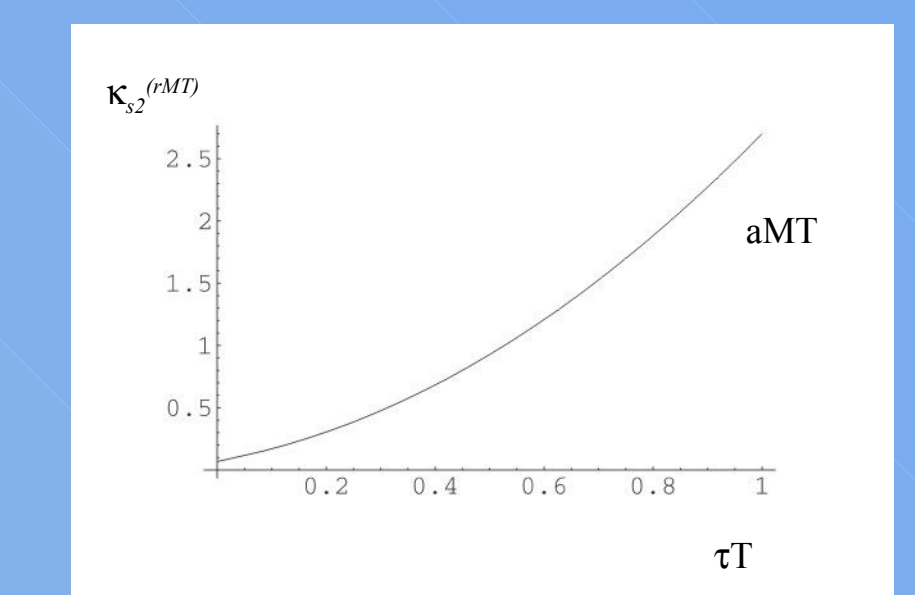
$$\omega(k) = \omega_0(k) \sqrt{1 + Re \Pi}$$

leads to the corrections in the sound velocity:

- from DOS and rMT diagrams in leading order (ω^0)
- aMT can be important in order ω^2 too.

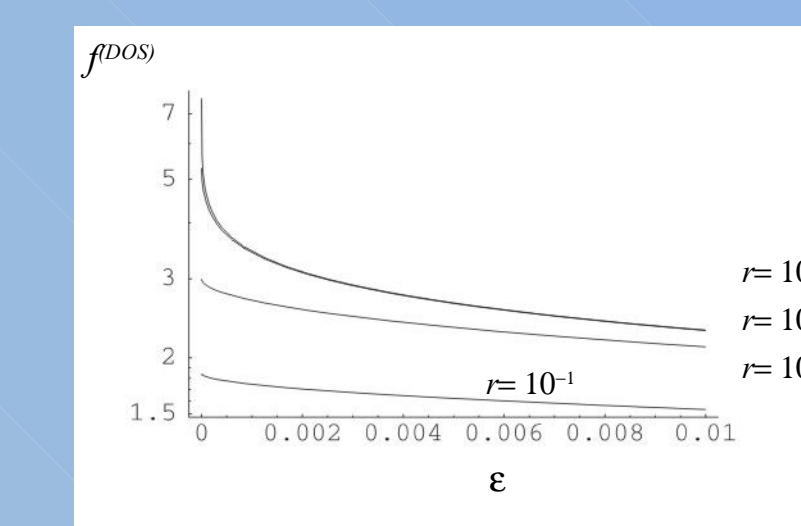
$$\frac{\Delta \omega^{\beta}(T, \omega)}{\omega} = \frac{\Delta v_s^{\beta}(T, \omega)}{v_n} \approx \frac{g^2 T^{\beta}}{d v_s^2} \left(\kappa_{att}^{\beta}(T, \tau) + \left(\frac{\omega^2}{T^2} \right) \kappa_{att}^{\beta}(T, \tau) \right) f^{\beta}(\epsilon, r, \gamma_{\phi})$$

The aMT coefficient:

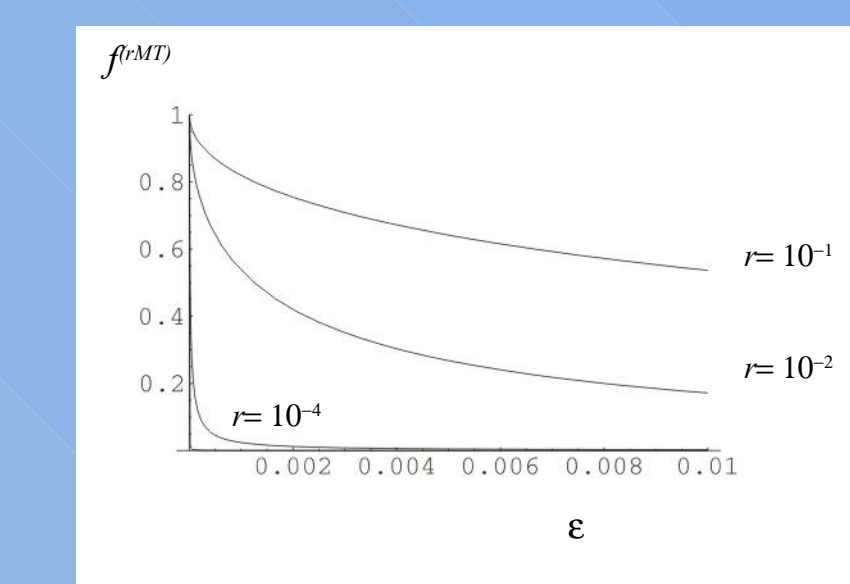


The temperature functions f^{β}

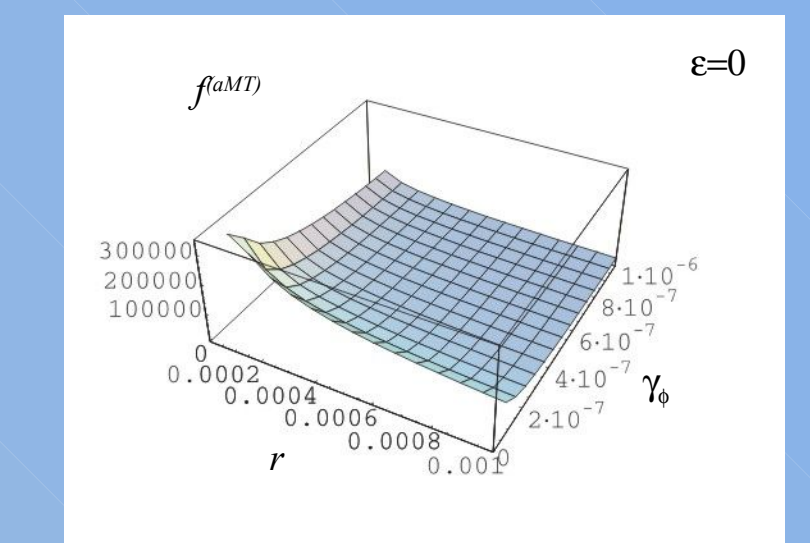
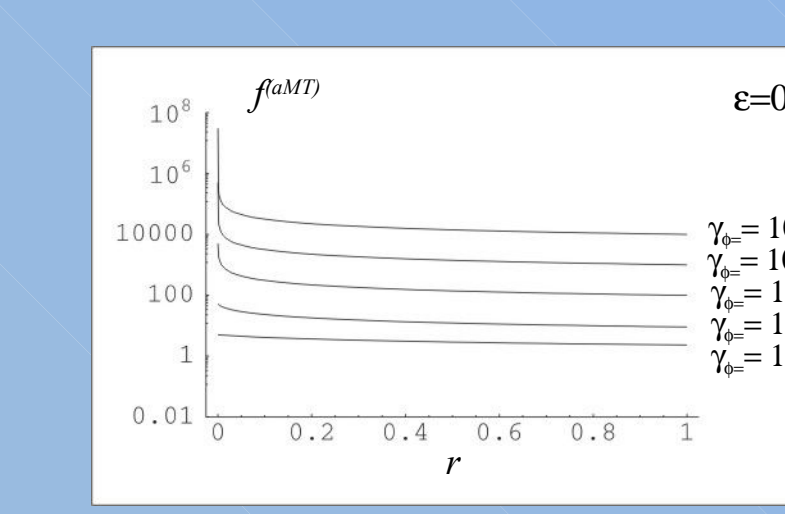
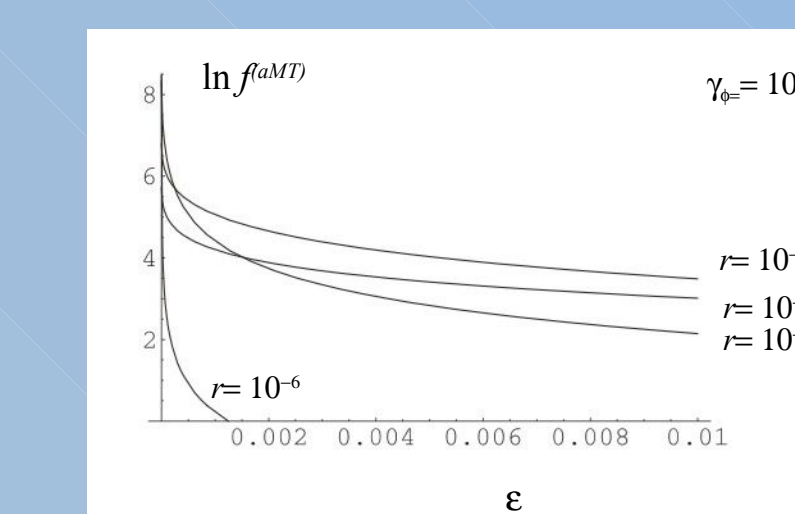
DOS



rMT



aMT



The temperature functions in aMT term at various values of parameters r and γ_{ϕ} .

VIII. Conclusion

- Although the fluctuation corrections to the phonon Green's function are given by the same diagrams as for the conductivity, the corresponding analytical expressions are different and lead to strikingly different results.
- No need for the impurity renormalization of the electron-phonon vertices.
- In the leading order, the result is independent on k .
- In quasi-2D case:
 - The AL term, (usually the largest contribution in conductivity), vanishes by symmetry for sound velocity and attenuation.
 - For the sound attenuation, all other diagrams (DOS, MT) are important.
 - For the velocity renormalization, the leading order (ω^0) is given by DOS+rMT, while aMT can be important in ω^2 order.
 - Contributions have different signs.
 - Phase-breaking must be included.
- Experimentally realizable even if effect smaller than conductivity (no AL) situation.