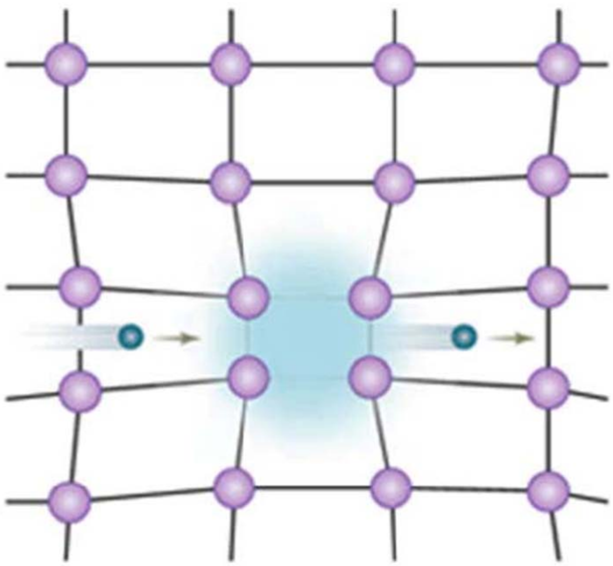
Superconductivity I
(Conventional Superconductors)

Jeff Sonier
Simon Fraser University



What is superconductivity?

Superconductivity was **discovered in 1911** by H. Kamerlingh Onnes at the University of Leiden, 3 years after he first liquefied helium.

This year marks the 100^{th} anniversary of superconductivity!

The Nobel Prize in Physics 1913

Heike Kamerlingh Onnes
(1853-1926)

Properties of a Superconductor

A superconductor is a material that exhibits both **perfect conductivity** and **perfect diamagnetism**.

Electrical Resistance in a Metal

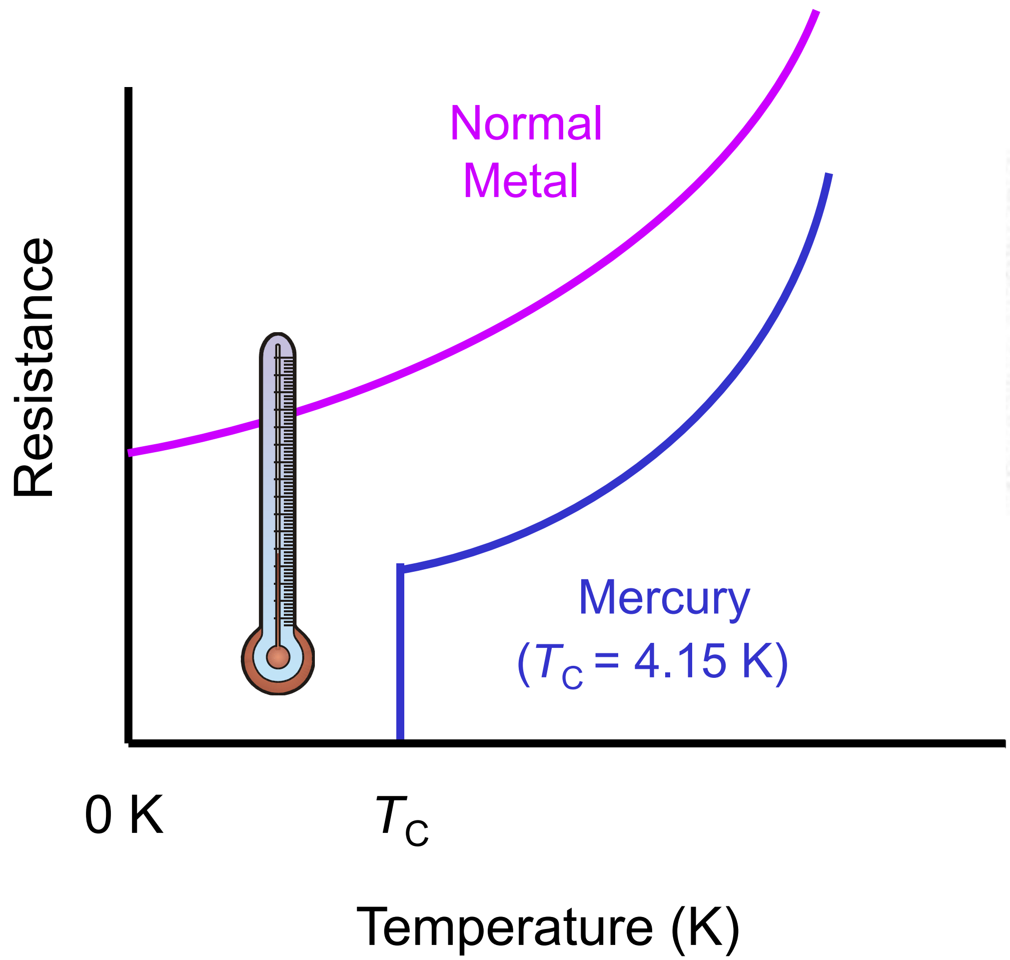
Resistance to the flow of electrical current is caused by ***scattering*** of electrons.

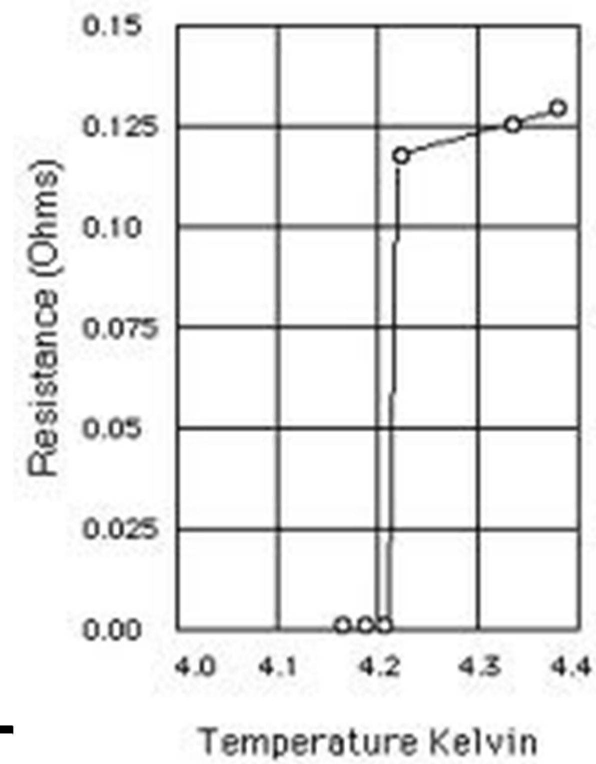
- scattering from lattice vibrations (phonons)
- scattering from defects and impurities
- scattering from electrons

Resistance causes ***losses*** in the transmission of electric power and ***heating*** that limits the amount of electric power that can be transmitted.

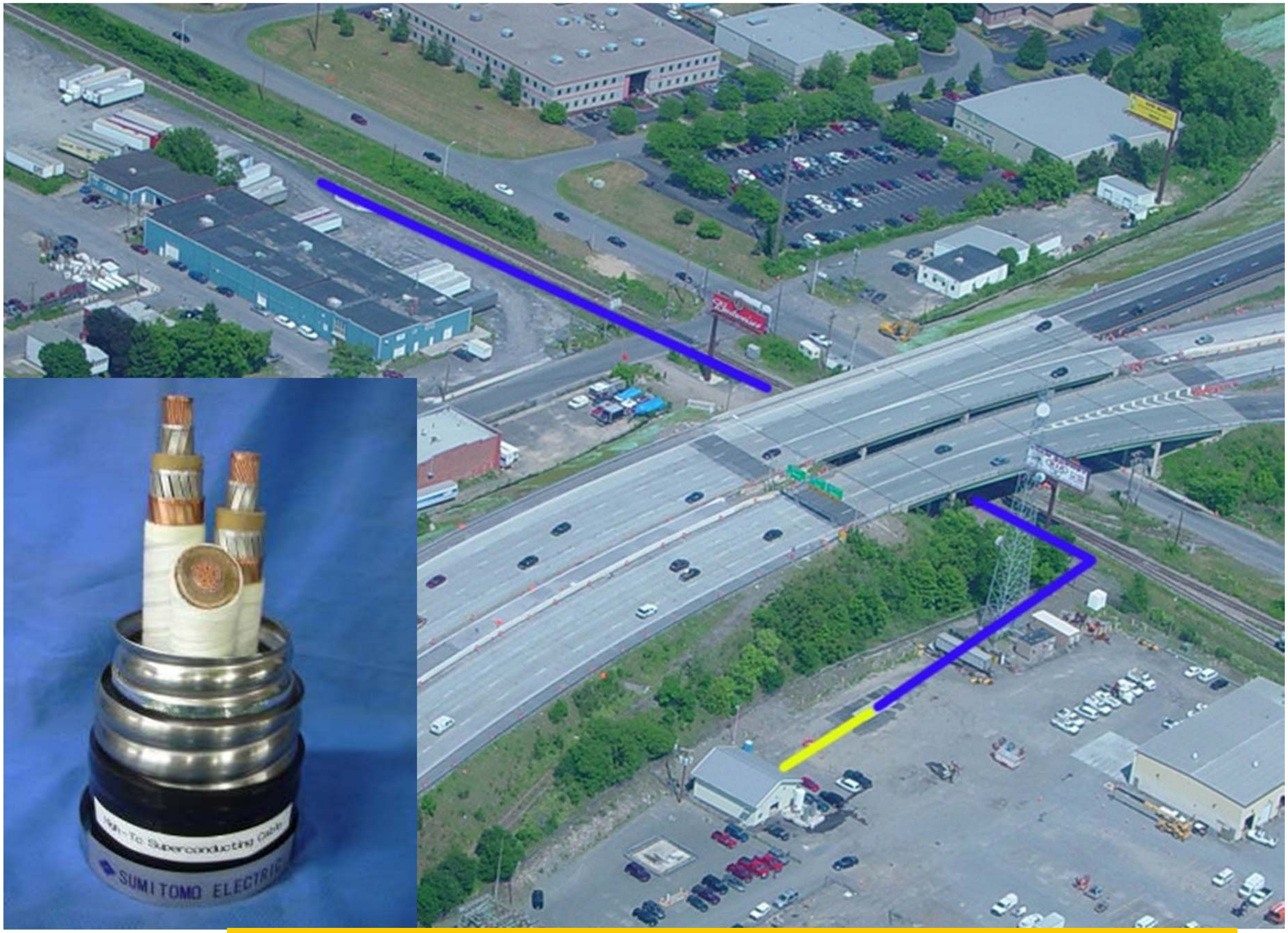
Zero Electrical Resistance

Onnes found that the electrical resistance of various metals (e.g. Hg, Pb, Sn) vanished below a **critical temperature** T_c.





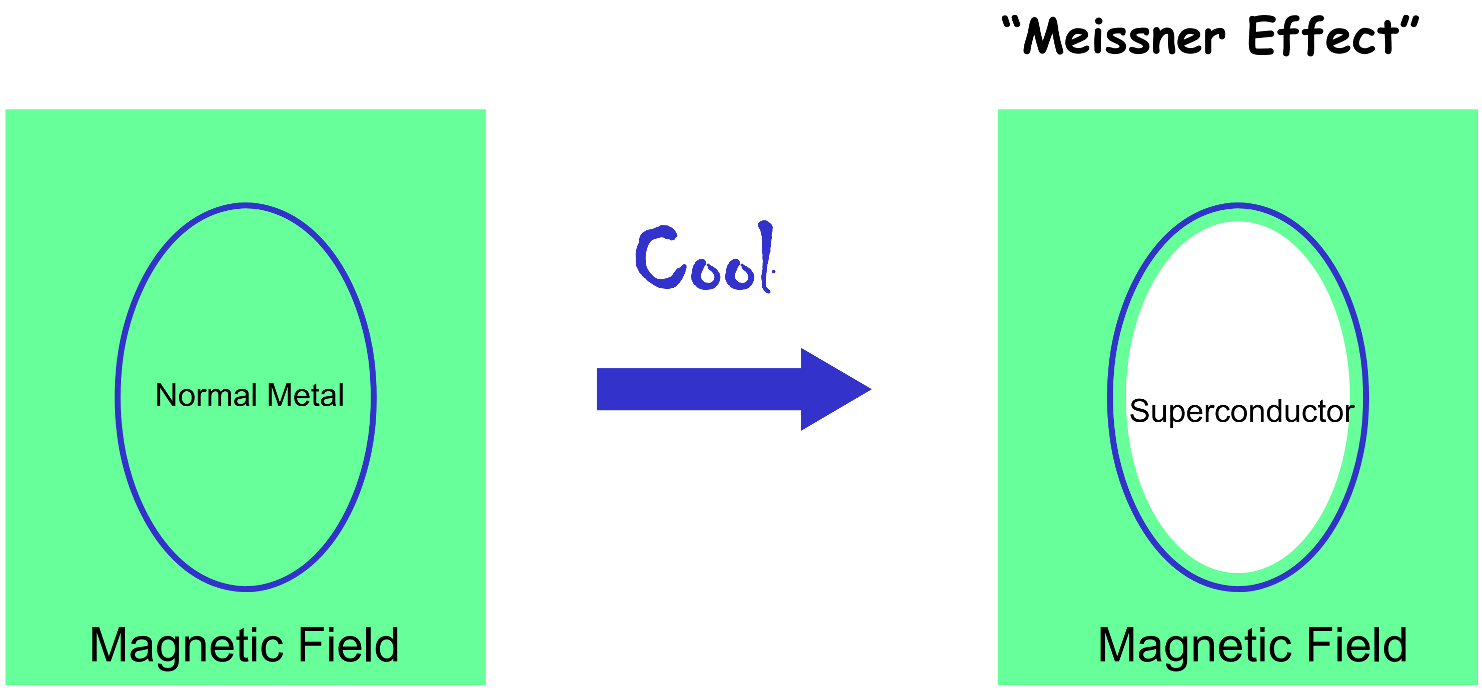
Superconducting Power Cable



Bi-2223 cable -Albany New York – commissioned fall 2006
February 2008 updated with YBCO section

Perfect Diamagnetism

In 1933, Meissner and Ochsenfeld discovered that magnetic field in a superconductor is **expelled** as it is cooled below T_c.



However, there is a limit as to how much field a superconductor can take! Superconductivity is destroyed above a **critical magnetic field** $H_c(T)$, separating the "normal" and superconducting states.

Superconducting Magnets

The absence of zero electrical resistance means that **persistent currents** flow in a superconducting ring. A major application of this property is **superconducting magnets**. With no energy dissipated as heat in the coil windings, these magnets are cheaper to operate and can sustain larger electric currents (and hence produce **greater magnet fields)** than electromagnets.

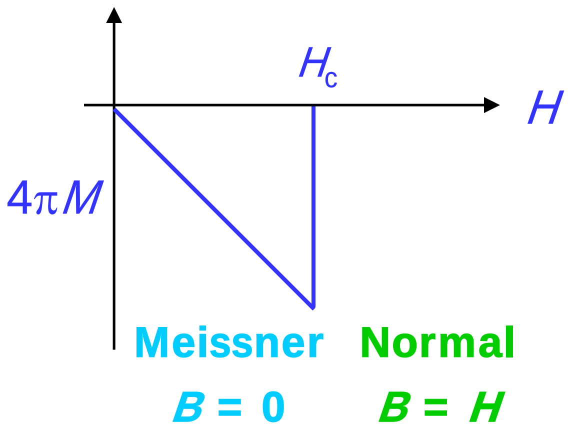
27 km Large Hadron Collider (LHC)

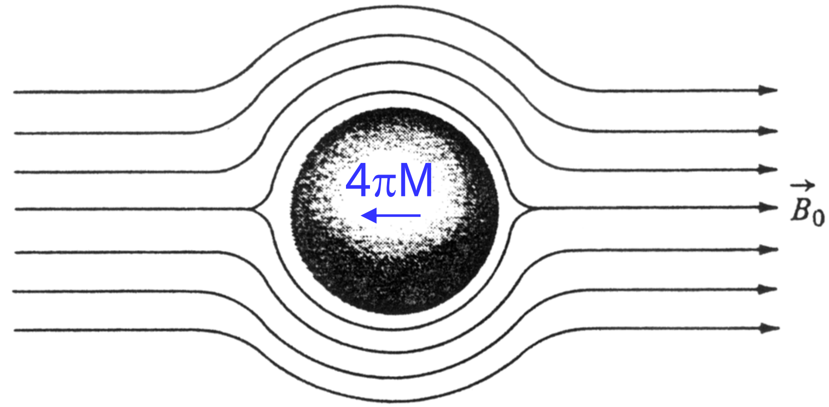
MRI machine

HELIOS

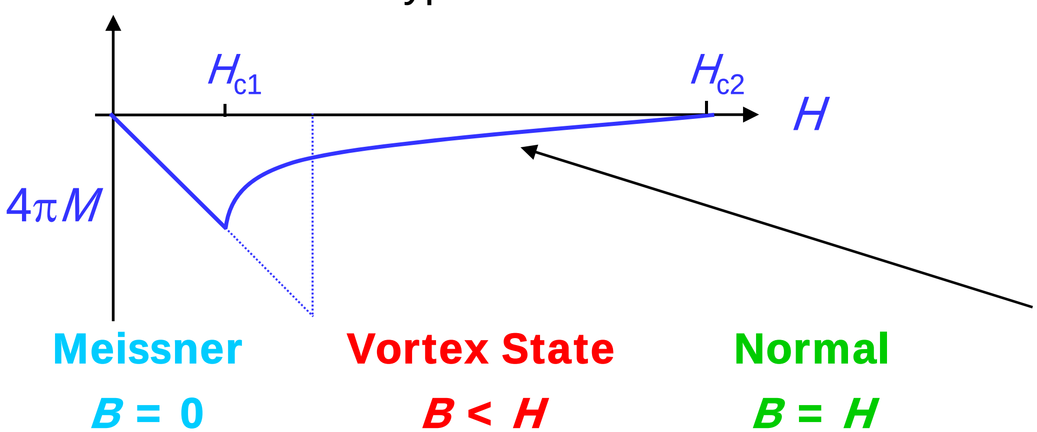
Magnetic Response of Type-I and Type-II Superconductors

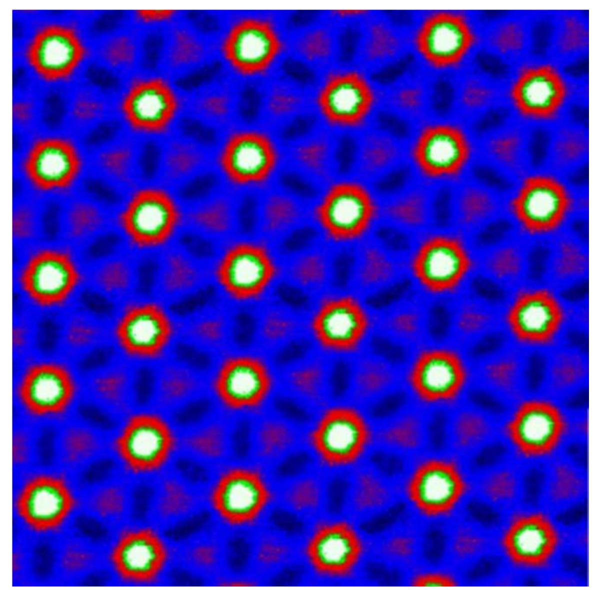
Type I





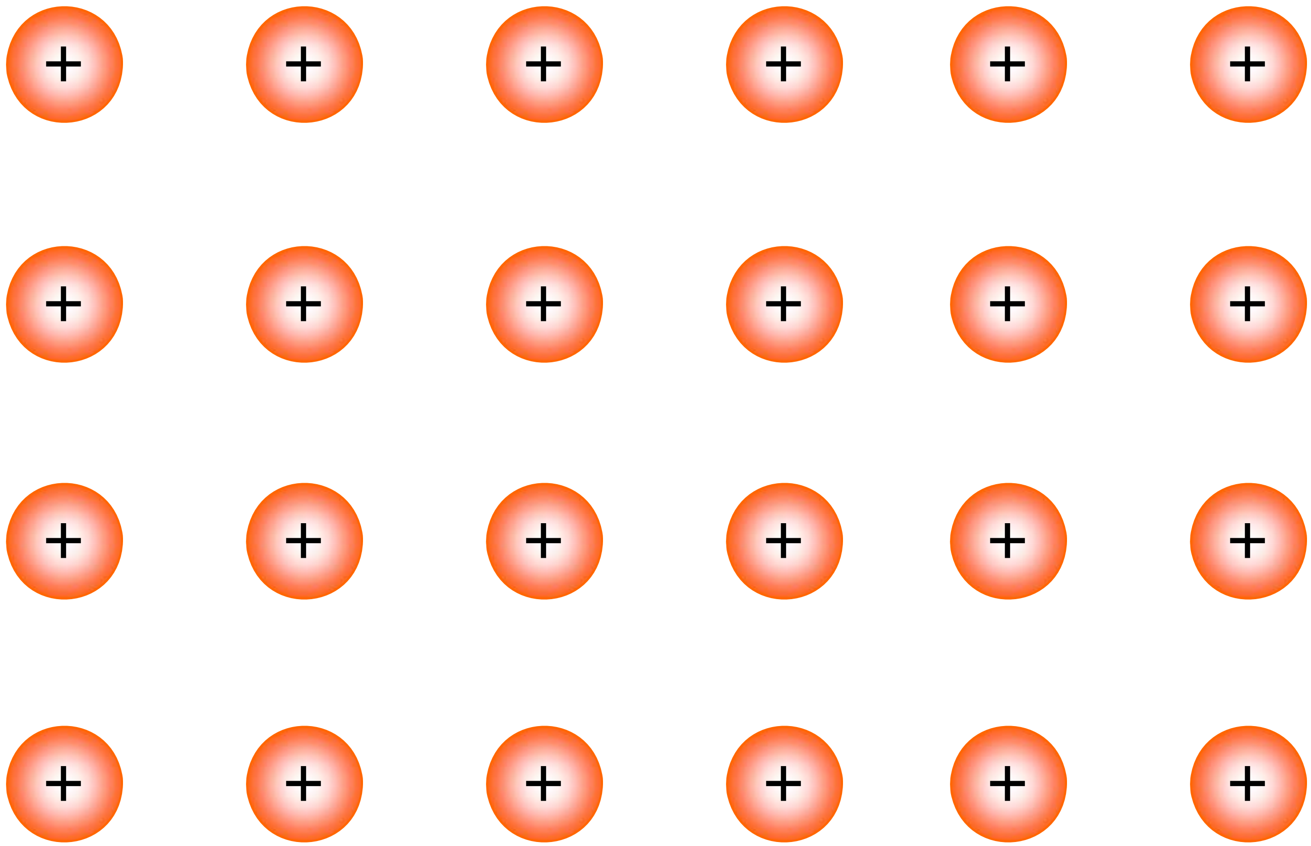
Type II





STM image of vortex lattice

The way to superconductivity…



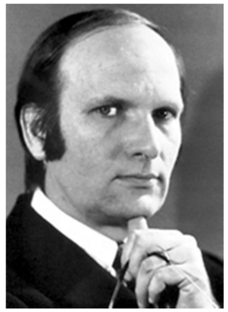
“Cooper pairs”: *pairs of electrons* caused by electron-phonon interaction

BCS Theory of Superconductivity

1972

J. Bardeen

L.N. Cooper



J.R. Schrieffer

General idea - Electrons pair up ("Cooper pairs") and form a ***coherent quantum state***, making it impossible to deflect the motion of one pair without involving all the others.

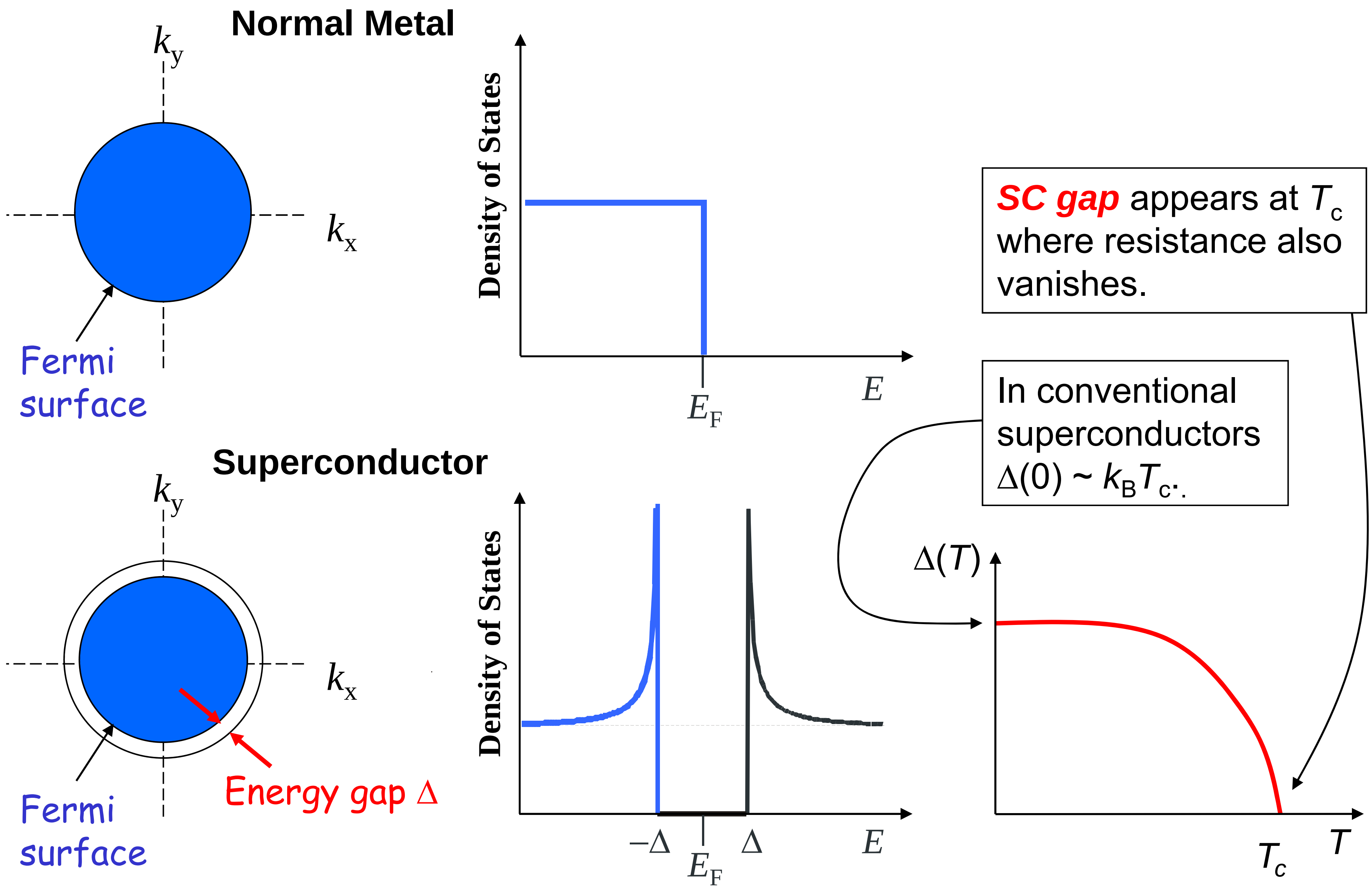
Zero resistance and the Meissner effect require that the Cooper pairs share the same phase $\Rightarrow$ **"quantum phase coherence"**

The BCS state is characterized by a complex ***macroscopic wave function***:

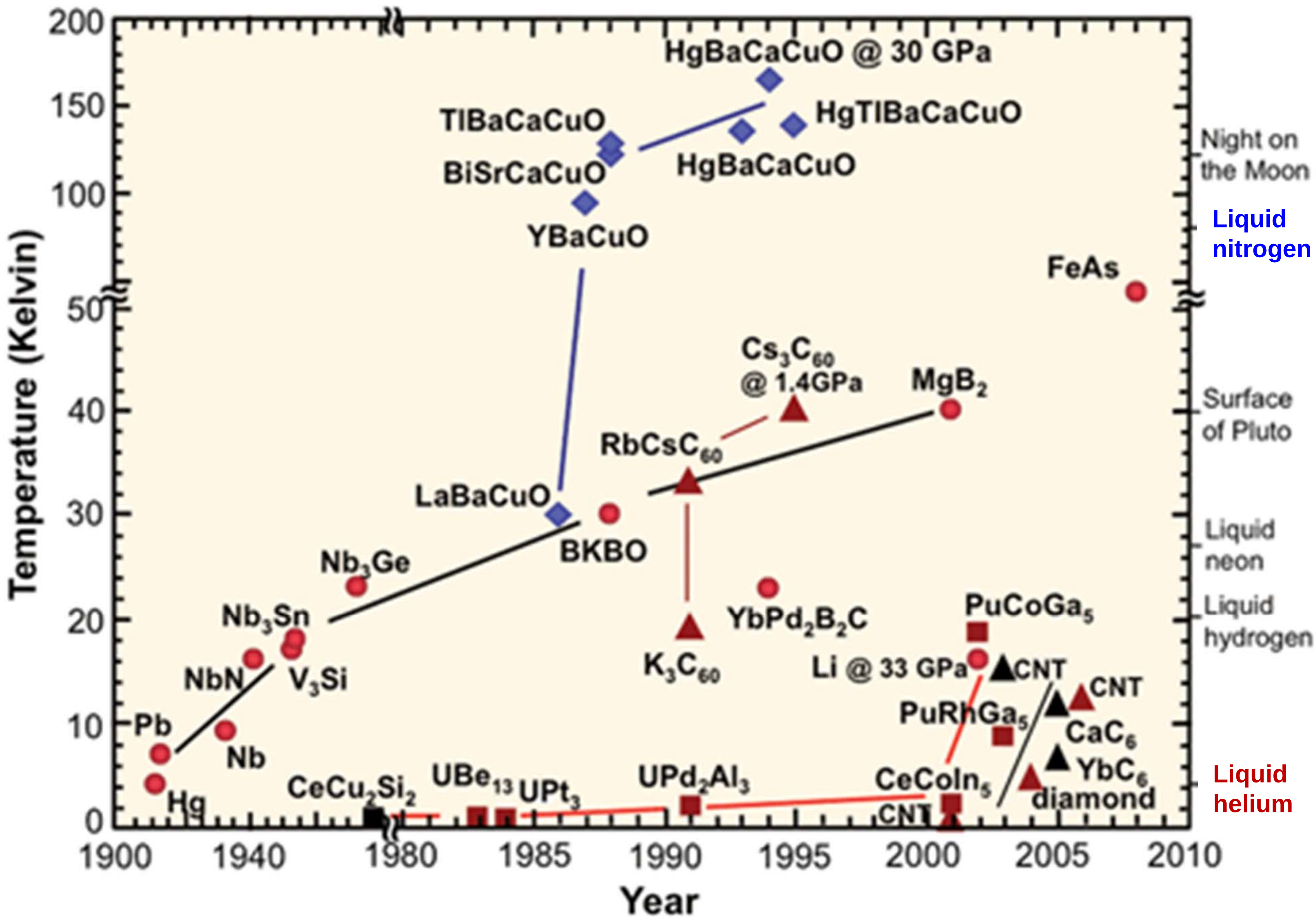
$$\Psi(\vec{r}) = \underbrace{\left|\Psi_0\right|}_{\text{Amplitude}} e^{i\underbrace{\theta(\vec{r})}_{\text{Phase}}}$$

Energy Gap

The pair binding manifests itself as an ***energy gap*** at the Fermi surface.



History of Superconductivity



Conventional superconductors conform to BCS or Bogoliubov theories.

The attractive interaction is usually mediated by **phonons**, and the pairing symmetry of the superconducting state is ***s*-wave**.

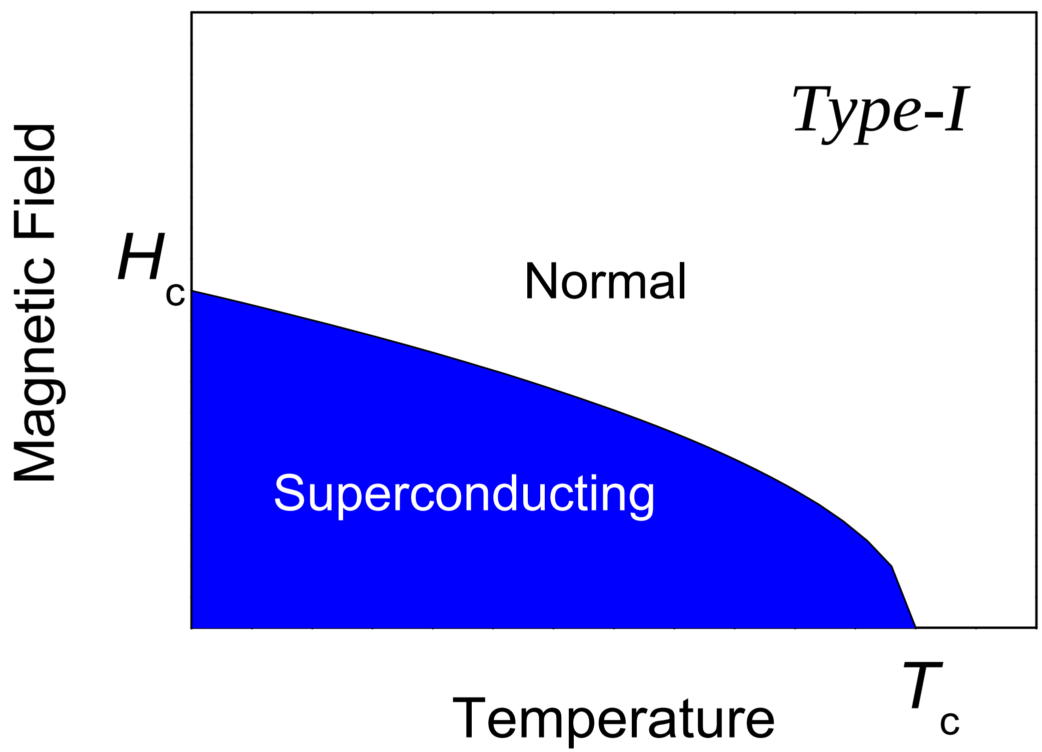
Some Applications of μSR to conventional low-T_c superconductors

- Intermediate state of a type-I superconductor
- Magnetic penetration depth λ in a polycrystalline sample
- Vortex core size from single crystal measurements

Type-I Superconductivity

A **type-I** superconductor is one in which $\lambda < \xi$, where λ is the **magnetic penetration depth** and ξ is the **coherence length** (more about these later).

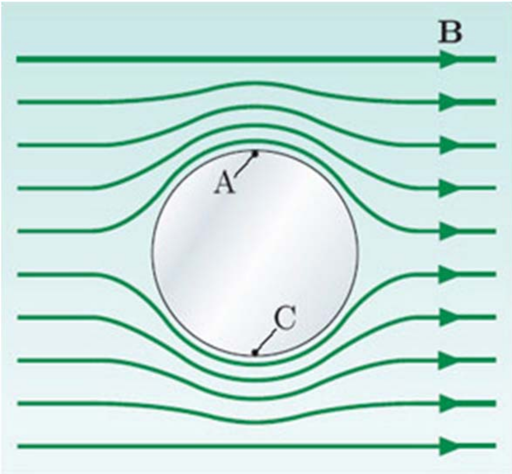
In these materials there is a **critical field** $H_c(T)$ below which the material is in the Meissner state, and above which superconductivity ceases.



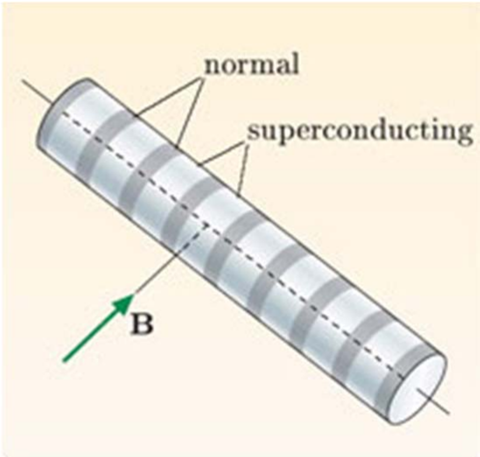
Intermediate State of a Type-I Superconductor

The magnetic response of a type-I superconductor depends on the **shape** of the sample.

The field at A and C reaches the critical field B_c at $B = B_c/2$.

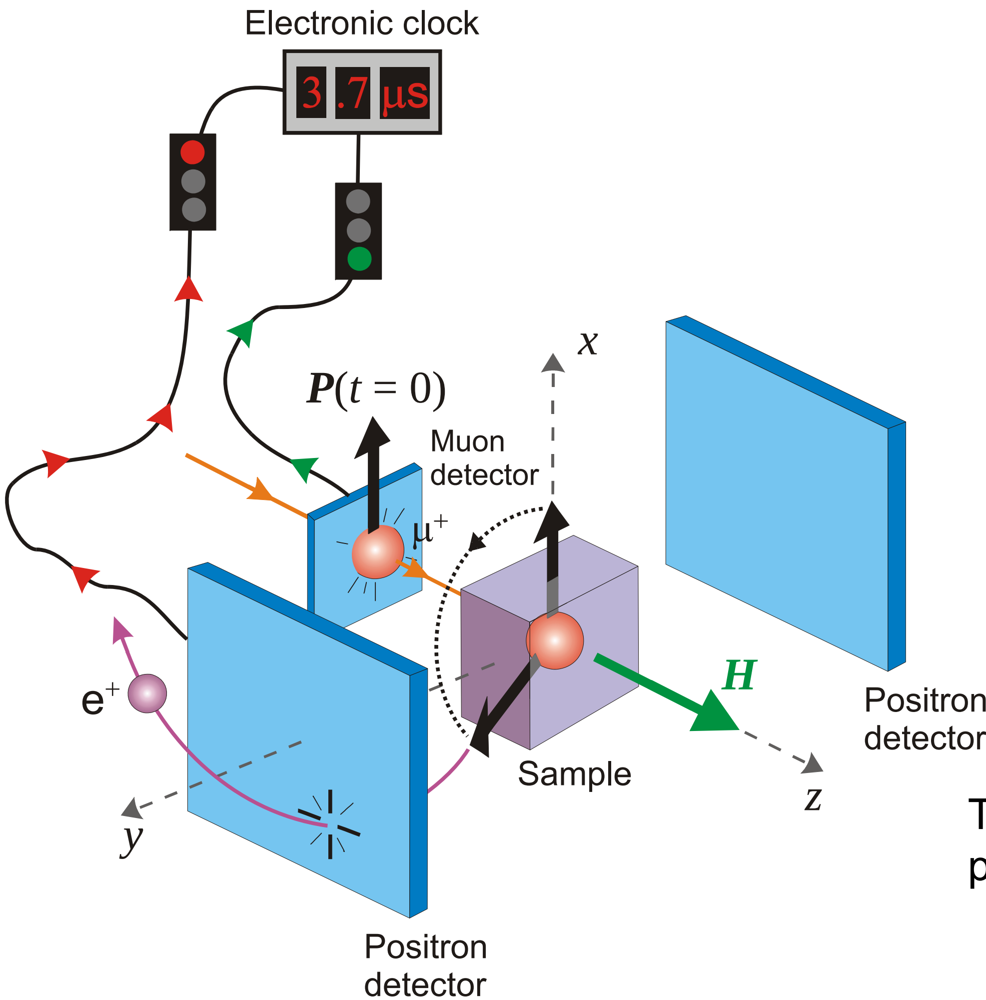


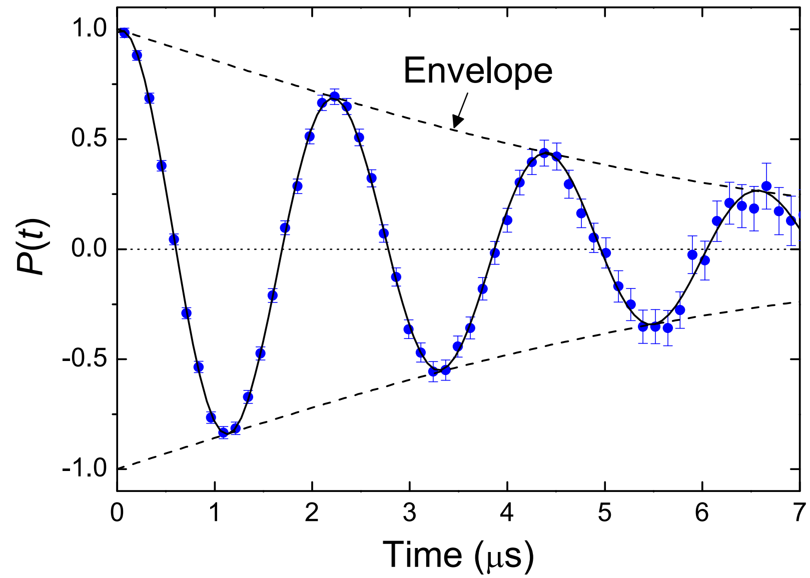
At $B_c/2 < B < B_c$, the cylinder splits up into small normal and superconducting regions. *i.e.* an **intermediate state**



Intermediate state in an Al plate

Transverse-Field μSR



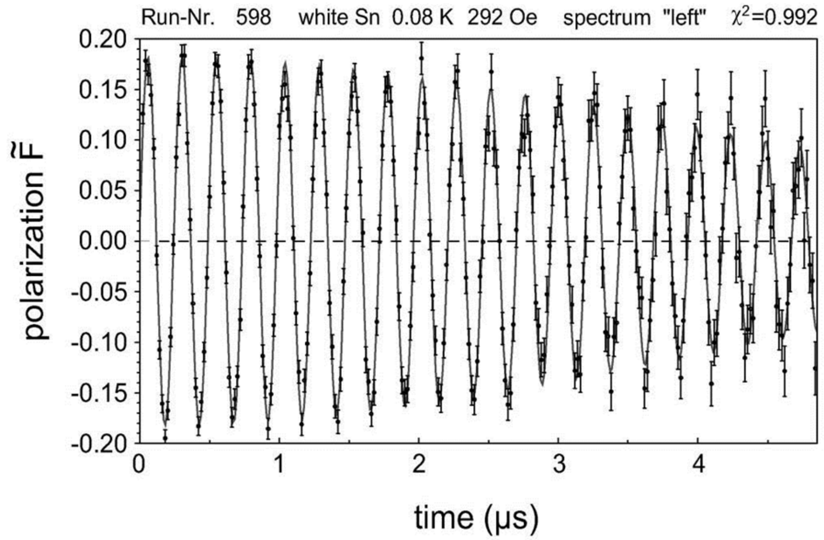


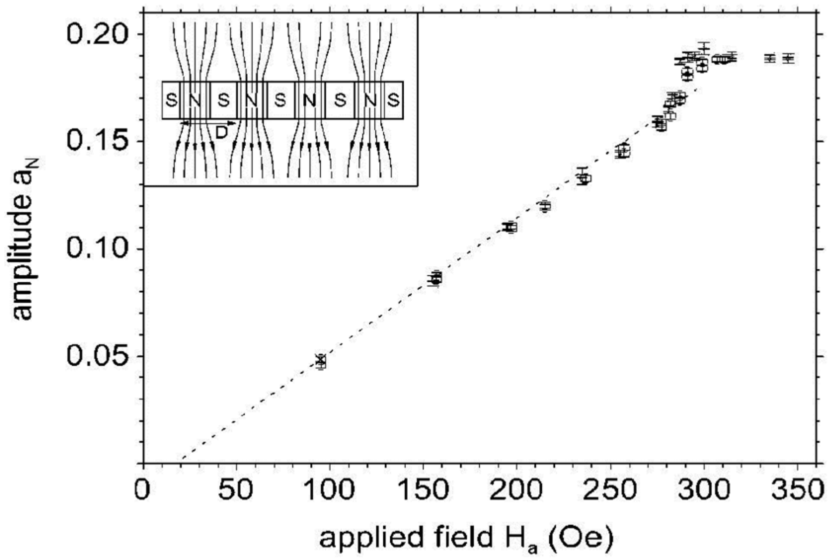
The time evolution of the muon spin polarization is described by:

$$P(t) = G(t)\cos(\gamma_\mu B_\mu t + \phi)$$

where $G(t)$ is a relaxation function describing the ***envelope*** of the TF-μSR signal.

Intermediate State of Single Crystal Sn Measured by TF-μSR





Muons stopping in the **N**ormal domains experience a local magnetic field ***B*** that is dominated by the contribution of the external field, and hence **coherently precess**.

$$A(t) = a_N e^{\sigma^2 t^2/2} \cos(\gamma_\mu Bt + \phi) + 0$$

Muons stopping in **S**uperconducting domains only experience the randomly oriented nuclear dipole fields, and hence do not coherently precess. Consequently, the superconducting volume fraction is a **"missing asymmetry"**.

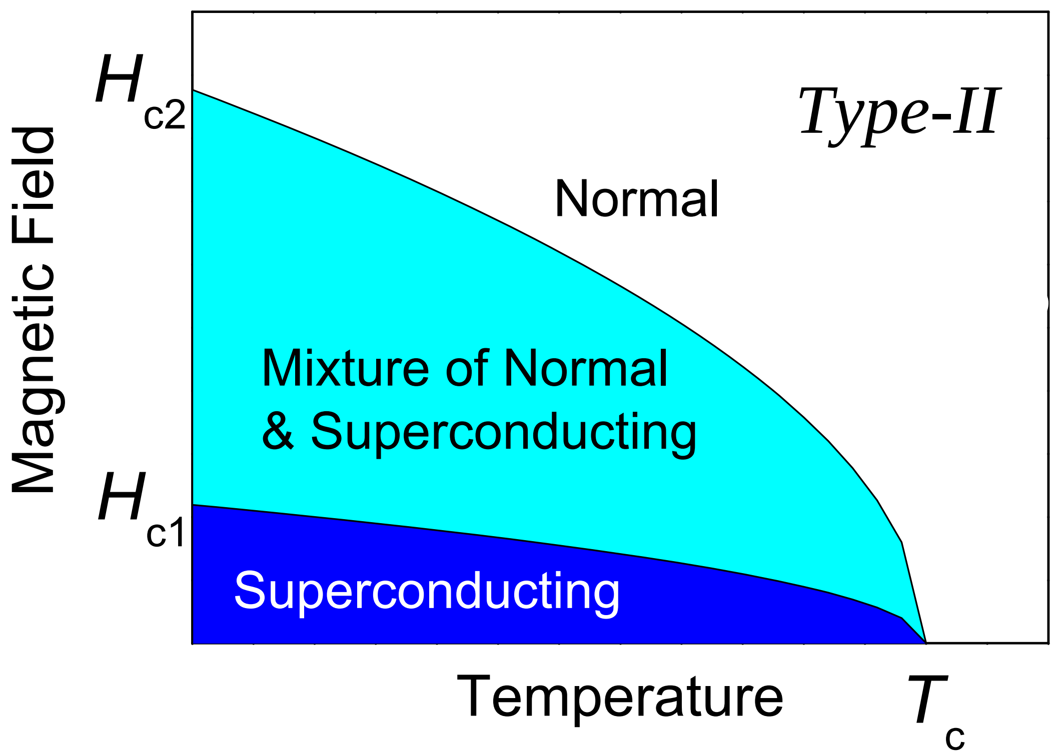
V.S. Egorov *et al*. Phys. Rev. B **64**, 024524 (2001)

Type-II Superconductivity

A **type-II** superconductor is one in which $\lambda > \xi$.

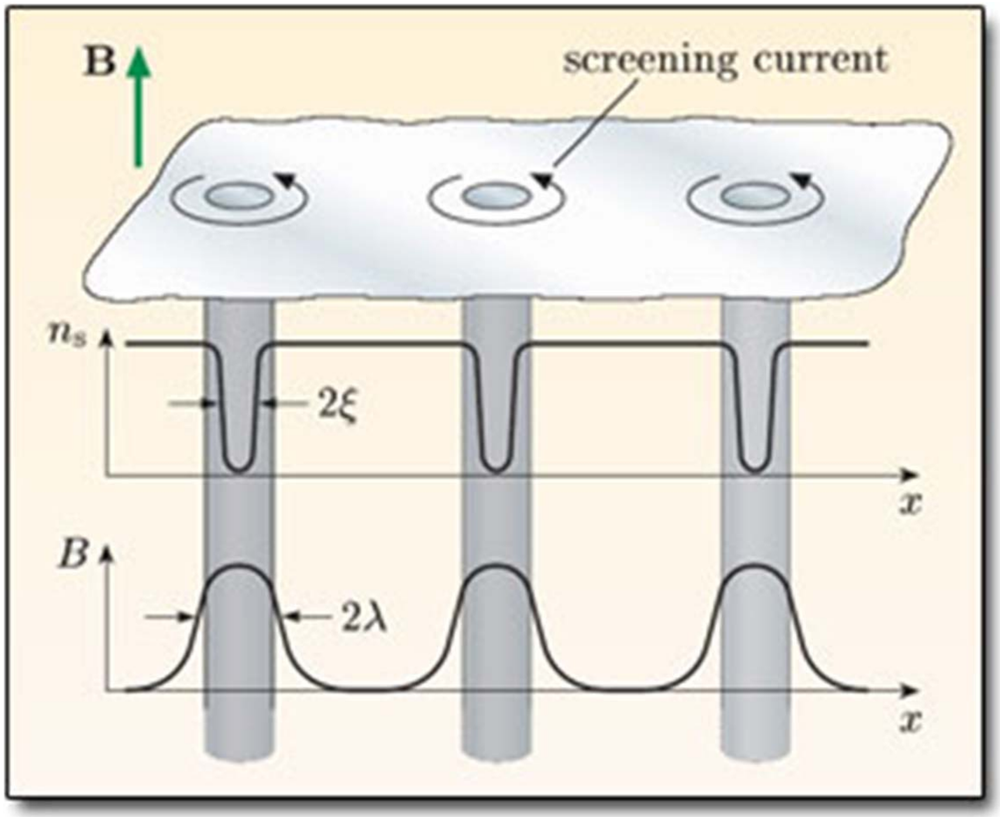
In these materials there is a phase that exists between **lower** and **upper** critical fields $H_{c1}(T)$ and $H_{c2}(T)$ where flux penetrates the material as a regular array of magnetic flux quanta, *i.e.* an **"Abrikosov vortex lattice"**.

At magnetic fields below $H_{c1}(T)$ the material is in the Meissner phase.



TF-μSR measurements in the vortex state

• deduce an **effective** ***magnetic penetration depth*** λ and superconducting ***coherence length*** ξ from the internal magnetic field distribution



$$\frac{1}{\lambda^2} = \frac{4\pi n_s e^2}{m^* c^2}$$

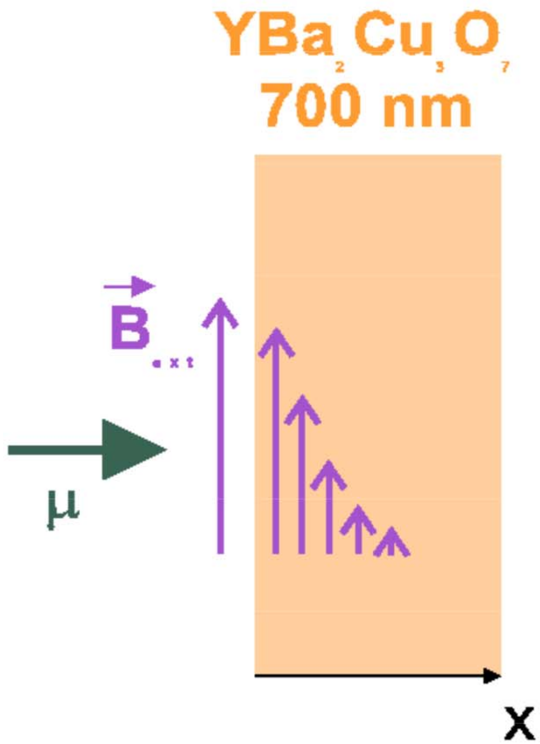
n_s ≡ **density of superconducting carriers**

ξ ~ **vortex core size**

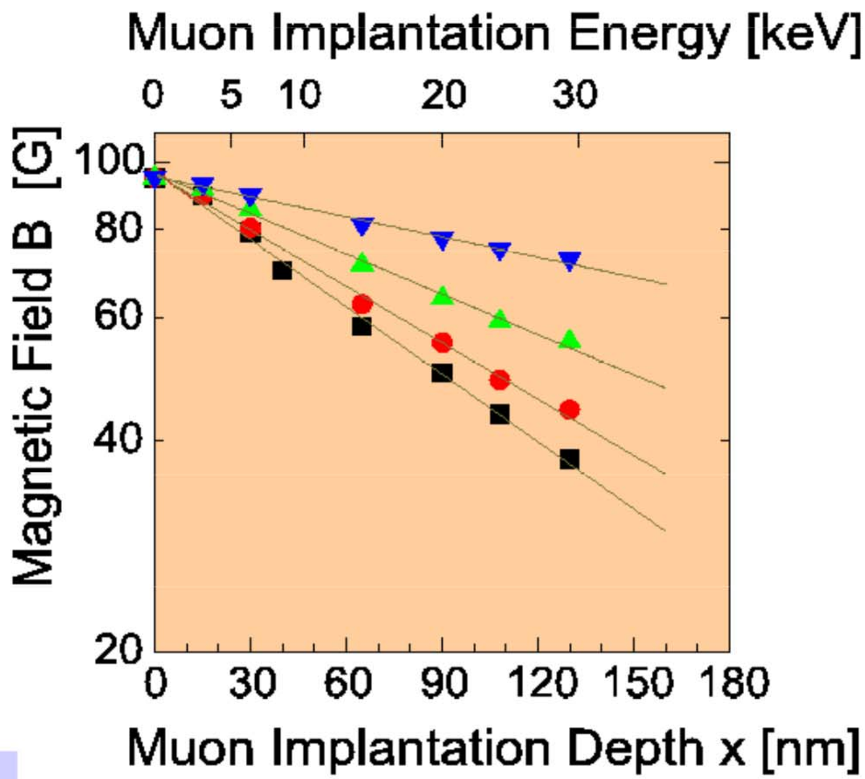
The length scale for **spatial variations of the superconducting order parameter** Δ(r), which decreases to zero at the vortex core centre, and rises to its maximum value outside the vortex core.

Direct measurement of the magnetic penetration depth in a high-Tc superconductor with low energy muons.

T.J. Jackson *et al.* Physical Review Letters **84**, 4958 (2000)



$$\vec{B}(x) = \vec{B}_{ext} e^{-\frac{x}{\lambda}}$$



The **time histogram** of decay positrons is given by:

$$N(t) = N(0)\exp(-t/\tau_\mu)[1 + A(t)]$$

where $A(t) = A_0 G(t)\cos(\omega t + \phi)$

In **polycrystalline** samples the measured internal magnetic field distribution $n(B)$ of the vortex lattice is nearly symmetric due to the random orientation of the crystallites with respect to the applied field. In this case it is generally sufficient to approximate the ***relaxation function*** $G(t)$ by a **simple Gaussian** depolarization function:

$$G(t) = \exp\left(-\frac{1}{2}\sigma^2 t^2\right)$$

which implies a Gaussian distribution of internal fields whose ***second moment***

$\left\langle (\Delta B)^2 \right\rangle = \left\langle (B - \langle B \rangle)^2 \right\rangle$ is related to σ via:

$$\sigma = \gamma_\mu \sqrt{\left\langle (\Delta B)^2 \right\rangle}$$

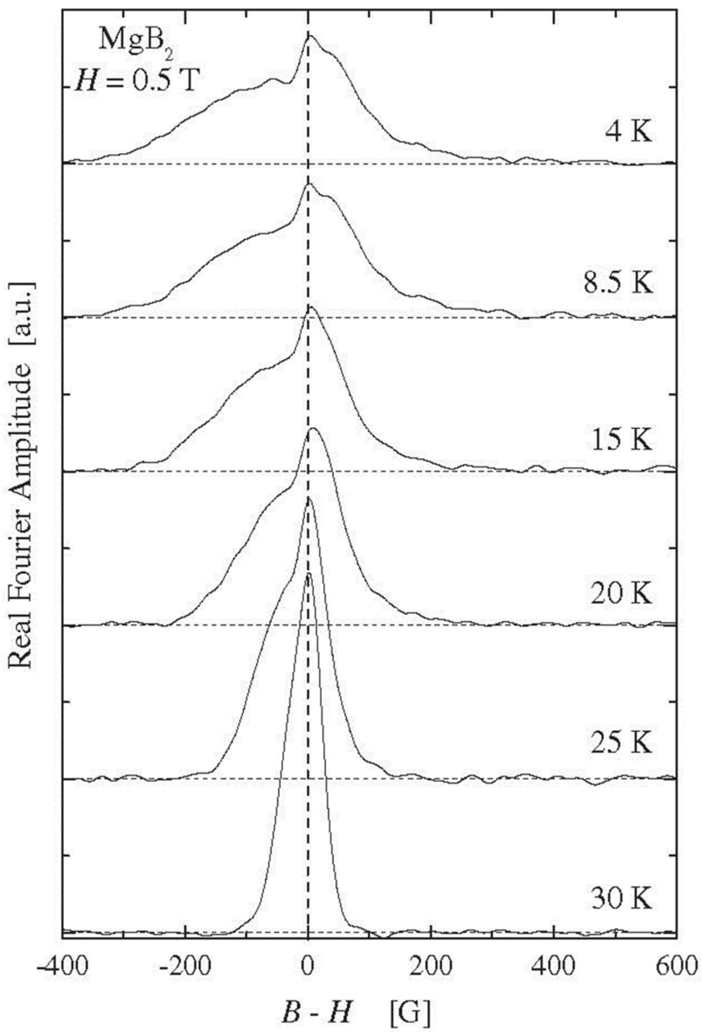
The second moment is a function of the **magnetic penetration depth** λ and **coherence length** ξ (*i.e.* two fundamental length scales). When $\lambda >> \xi$ and $H >> H_{c1}$ the following relation is generally obeyed:

$$\boxed{\sigma \propto \sqrt{(\Delta B)^2} \propto \frac{1}{\lambda^2} \propto \frac{n_s}{m^*}}$$

Density of superconducting carriers

Polycrystalline MgB_2 (T_c = 38.5 K)

FFTs of TFμ-SR signal at H = 0.5 T



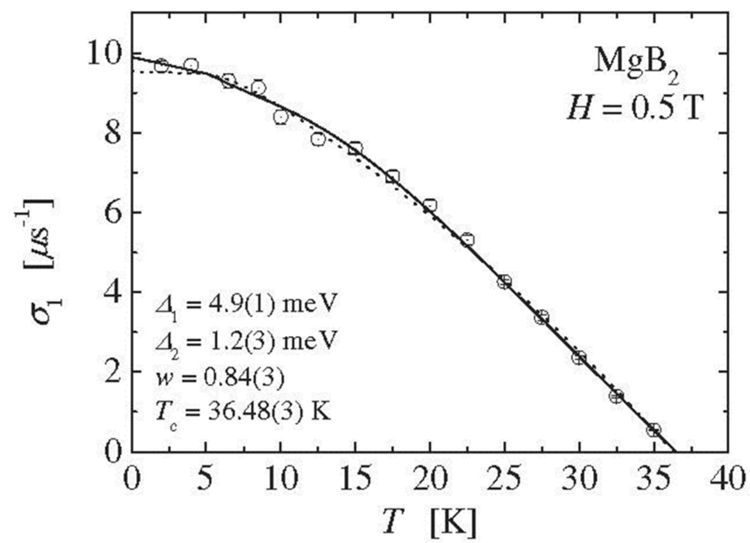
K. Ohishi *et al.* J. Phys. Soc. Jpn. **72**, 29 (2003)

The TF-μSR signal in the **time domain** is fit to the following relation:

$$A(t) = A_s e^{-\sigma^2 t^2/2}\cos(\gamma_\mu B t + \phi_s) + A_b e^{-\sigma_b{}^2 t^2/2}\cos(\gamma_\mu H t + \phi_b)$$

where the depolarization rate is related to the ***second moment*** of the internal field distribution $n(B)$ as follows:

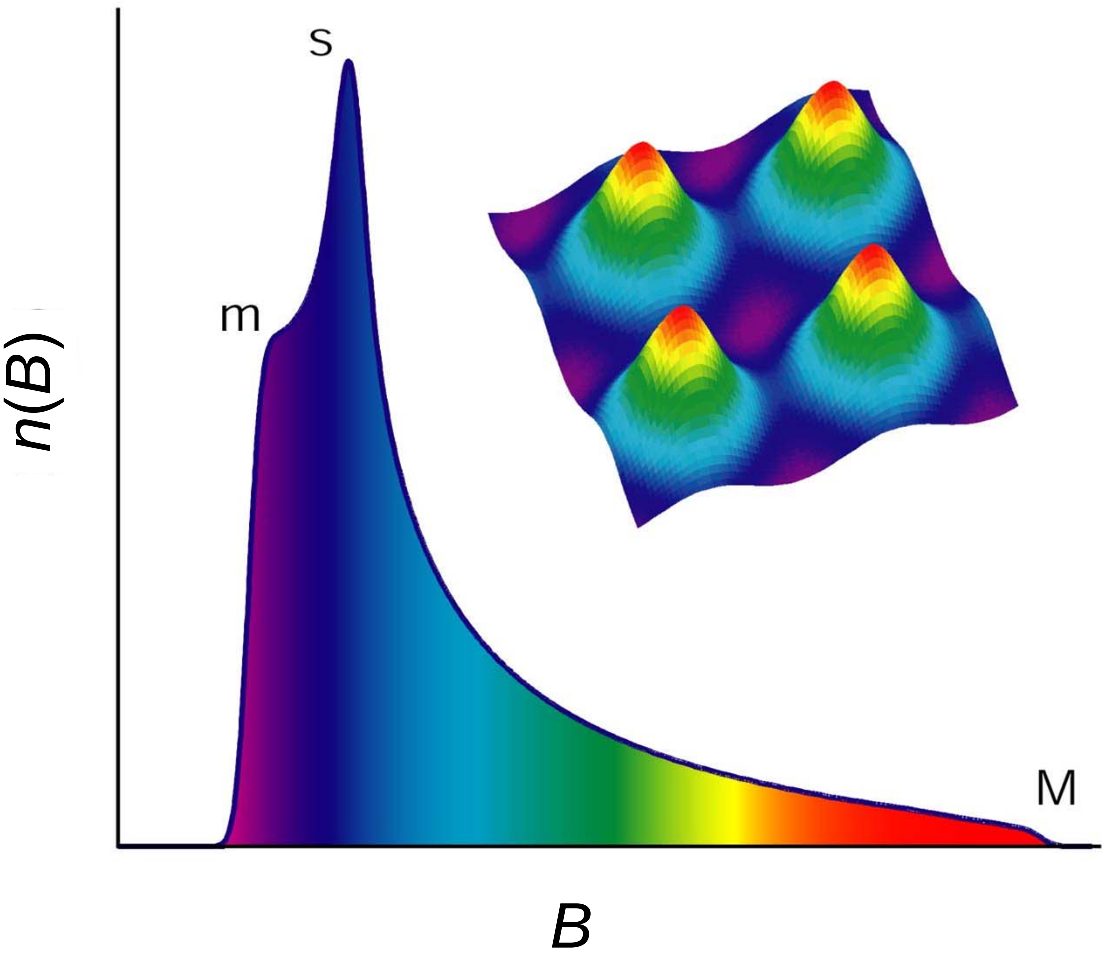
$$\sigma = \gamma_\mu \sqrt{\langle(\Delta B)^2\rangle}$$

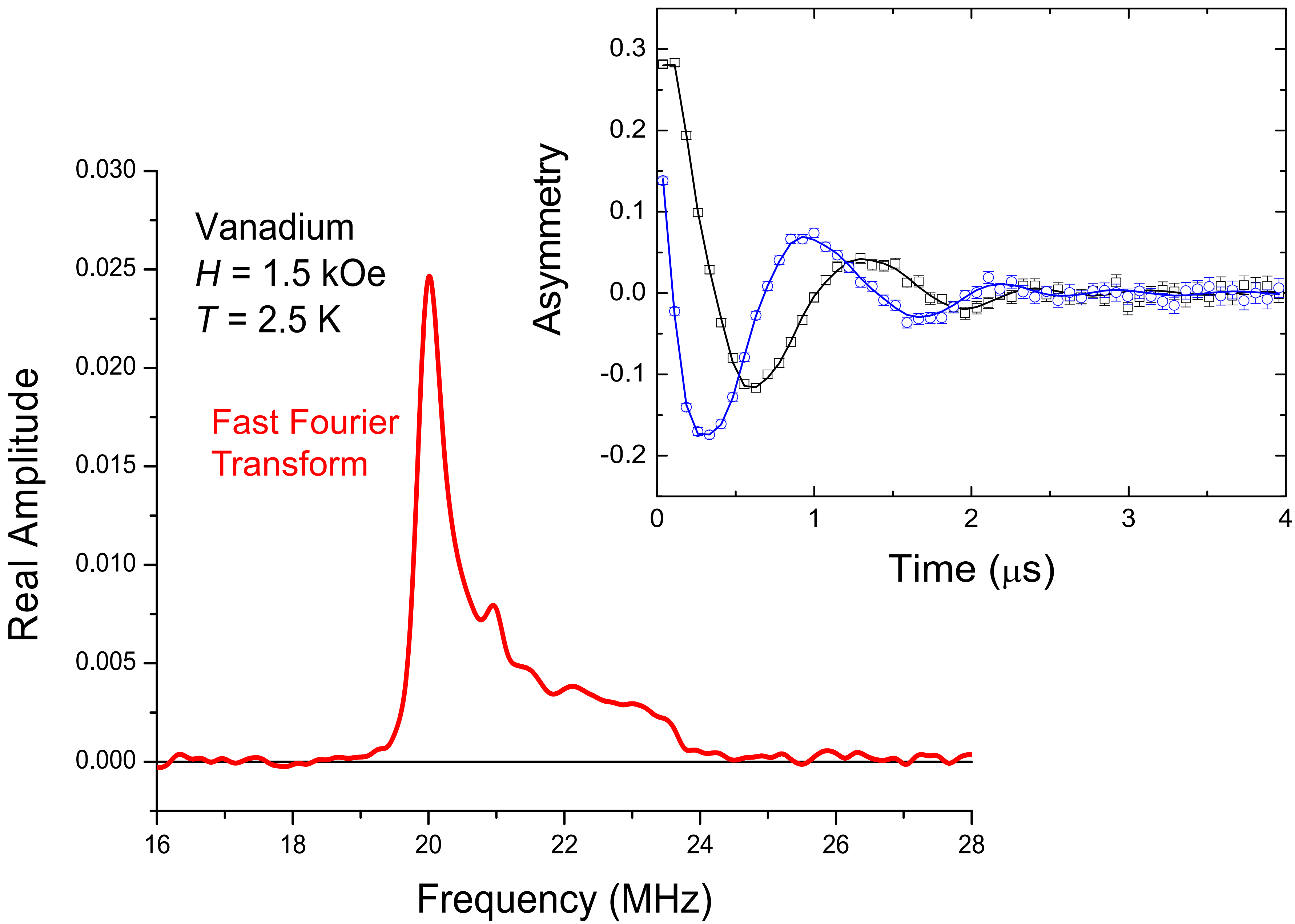


Vortex Lattice of a Type-II Superconductor

TF-μSR is ideally suited to measure the **internal magnetic field distribution**

e.g. field distribution of a square vortex lattice (from Mitrovic *et al.*)





0.030
0.025
0.020
0.015
0.010
0.005
0.000
Real Amplitude
Vanadium
H = 1.5 kOe
T = 2.5 K
Fast Fourier
Transform
16
18
20
22
24
26
28
Frequency (MHz)
0.3
0.2
0.1
0.0
-0.1
-0.2
Asymmetry
0
1
2
3
4
Time (μs)

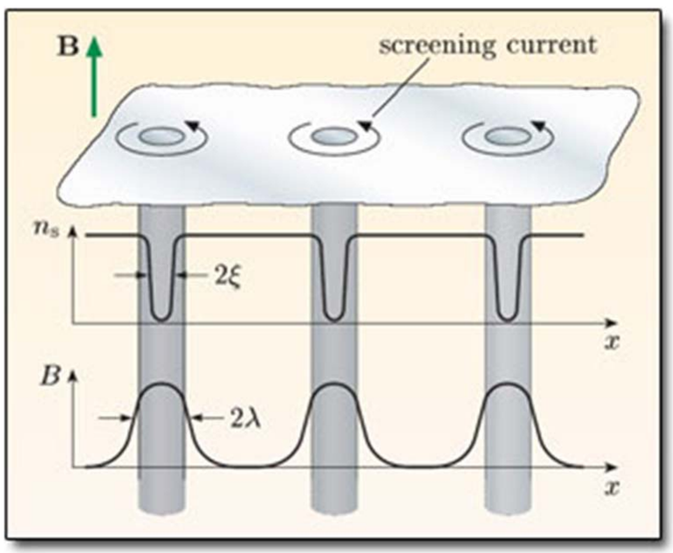
Fit of Single Crystal TF-μSR Spectra to Phenomenological Model

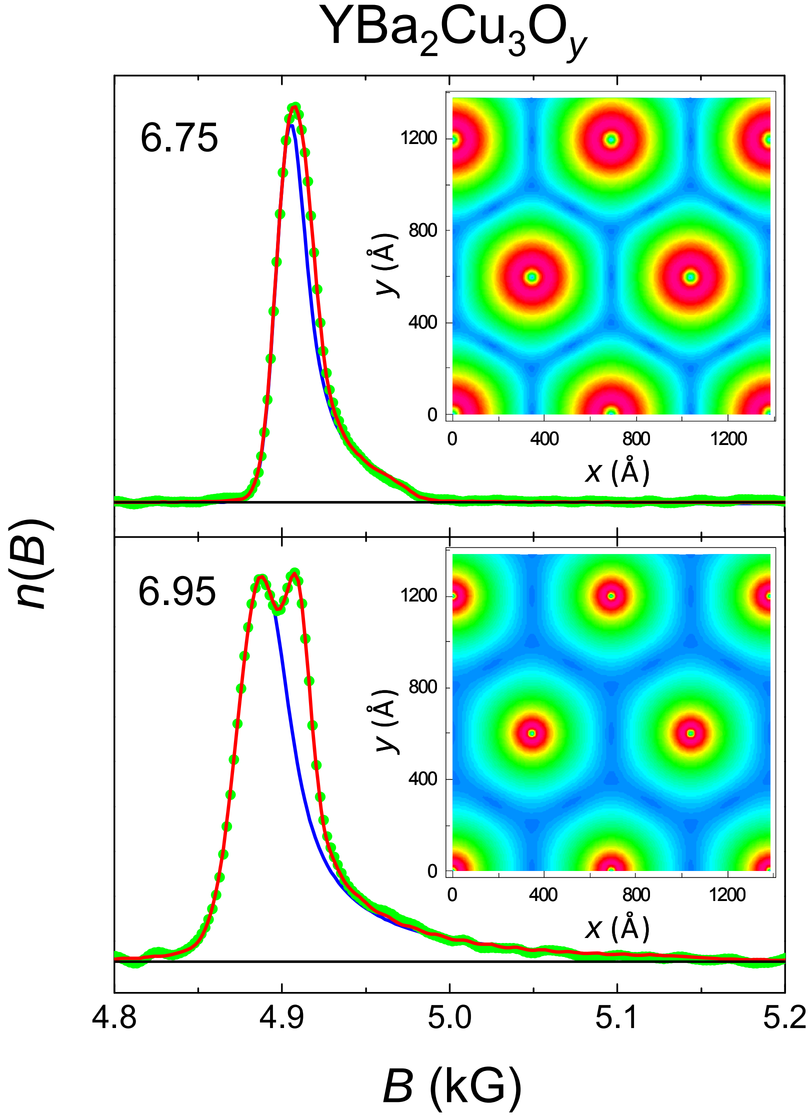
GL-model of spatial field profile:

$$B(\vec{r}) = B_0(1-b^4)\sum_{\vec{G}} \frac{uK_1(u)e^{-i\vec{G}\cdot\vec{r}}}{\lambda_{ab}^{\ 2}G^2}$$

$$b = \frac{B}{B_{c2}} \qquad u^2 = 2\xi_{ab}G^2(1+b^4)[1-2b(1-b)^2]$$

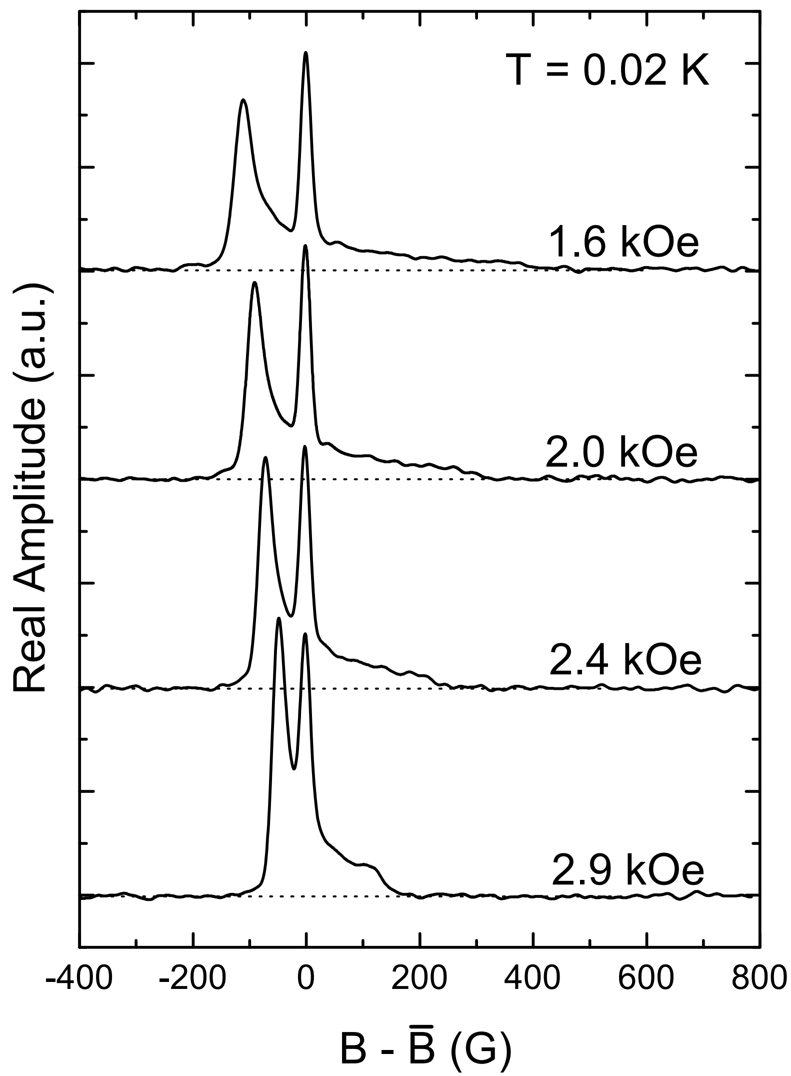
$\lambda_{ab} \equiv$ effective penetration depth
$\xi_{ab} \equiv$ effective vortex core size

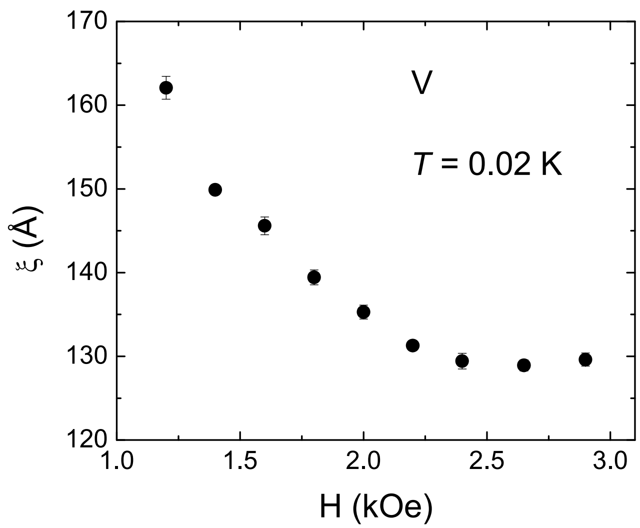


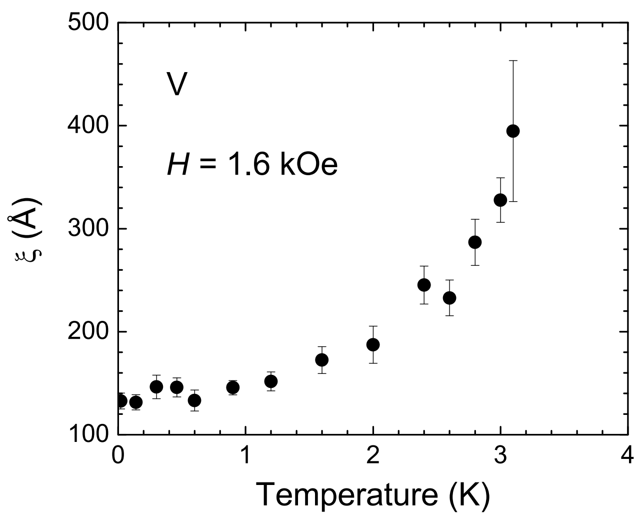


JES *et al.*, Phys. Rev. B **76**, 134518 (2007)

Pure Vanadium



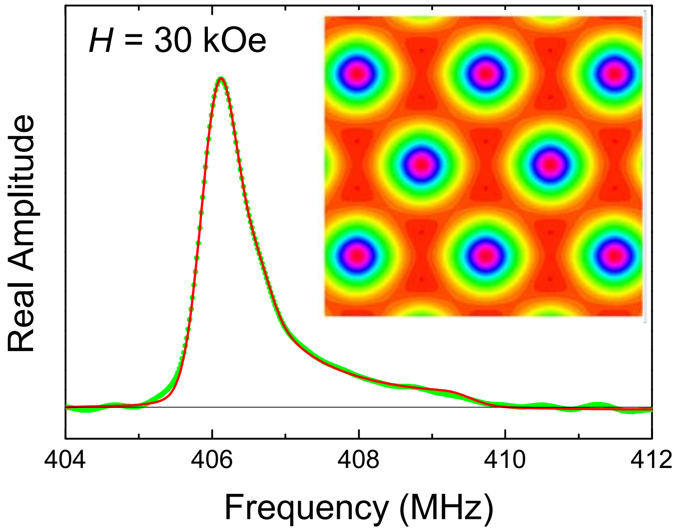


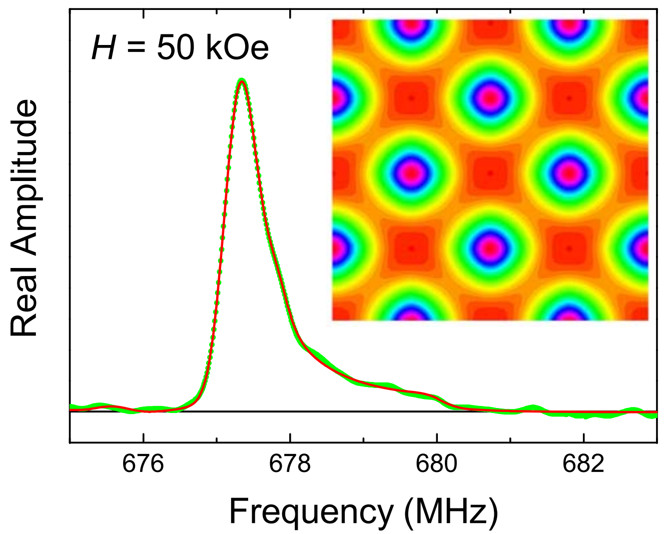


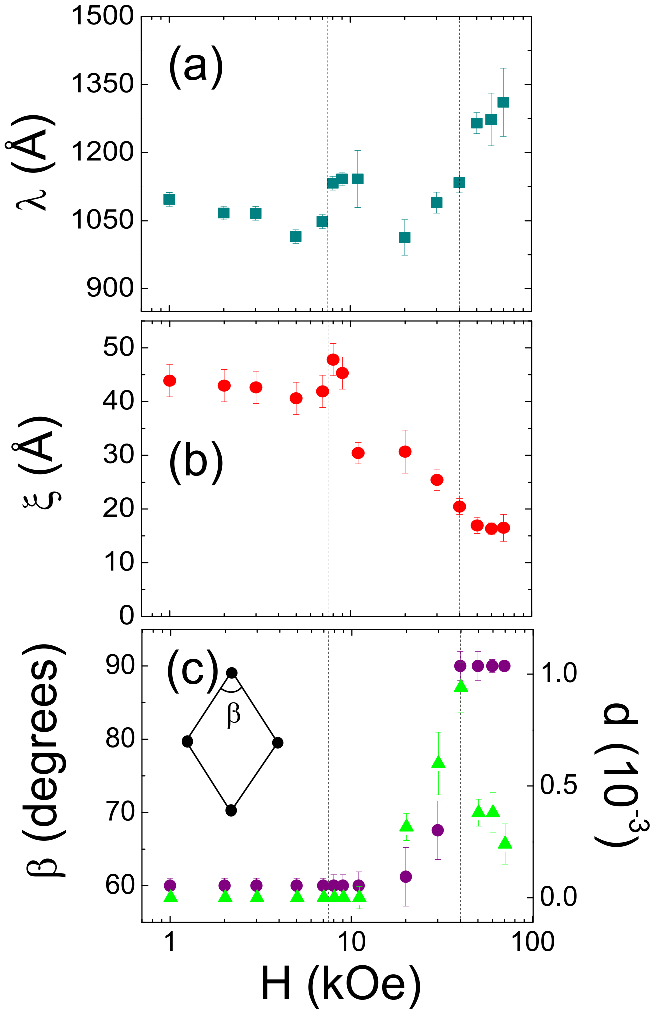
M. Laulajainen *et al*. Phys. Rev. B **74**, 054511 (2006)

Vortex lattice of V_3Si (T_c = 17 K)

TF-μSR line shapes at T = 3.8 K



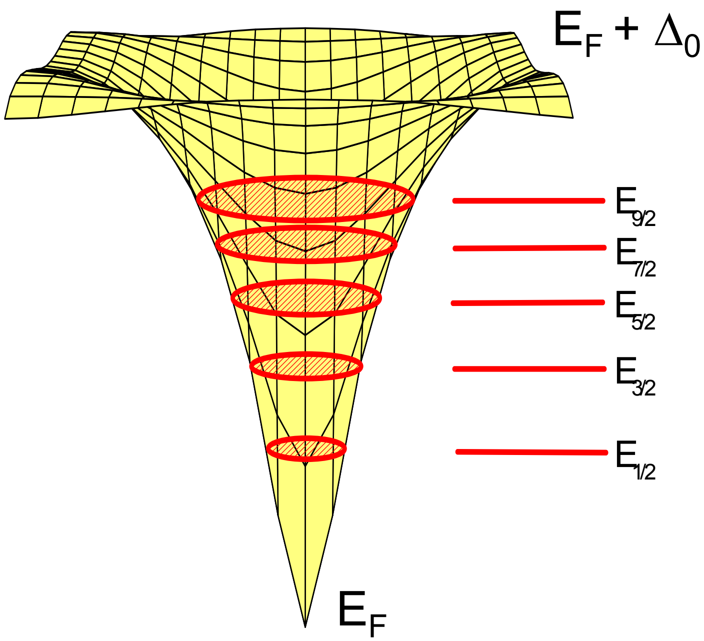




JES *et al.*, Phys. Rev. Lett. **93**, 017002 (2004)

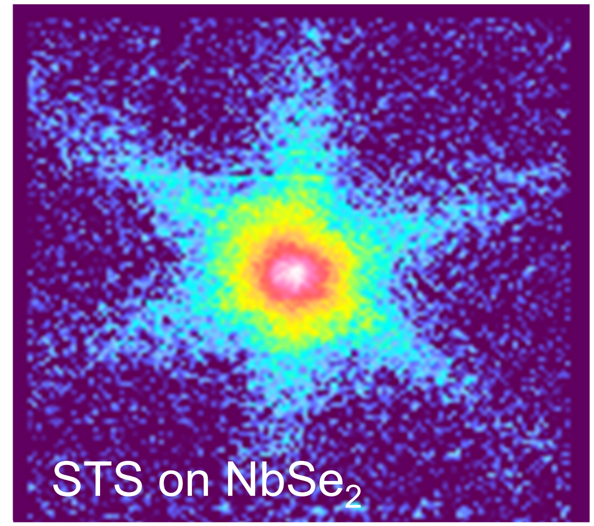
Vortex Electronic Structure

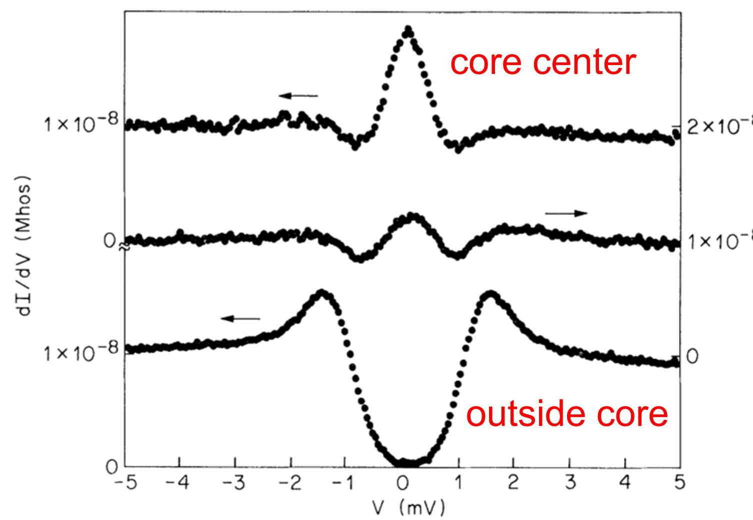
Rapid spatial variation of the pair potential $\Delta(r)$ at the vortex site



Low-lying quasiparticle excitations are **bound** to the vortex core.

First observation of bound core states



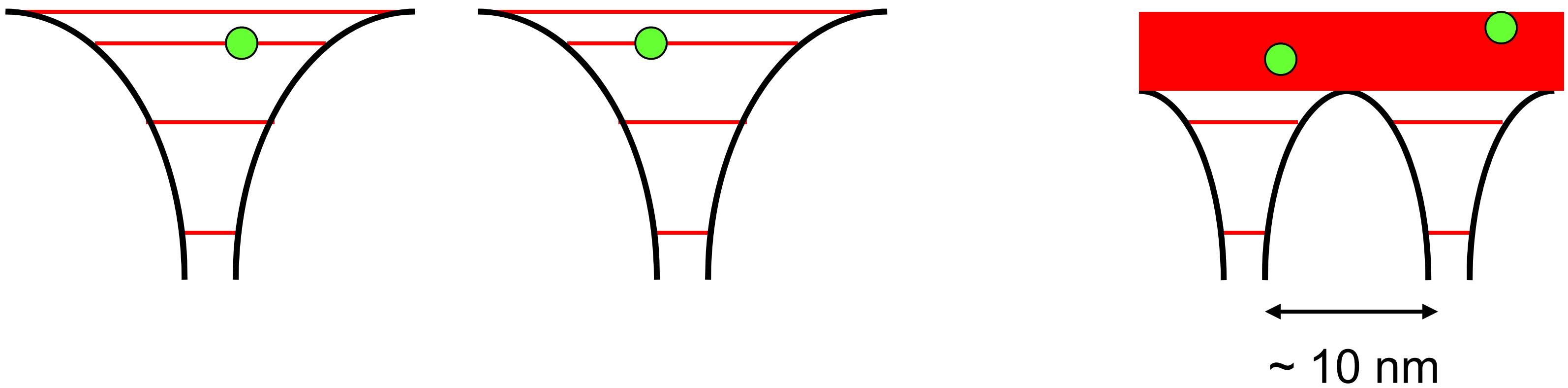


Hess *et al*. PRL **62**, 214 (1989)

Conduction by Electrons in Solids

Atoms far apart

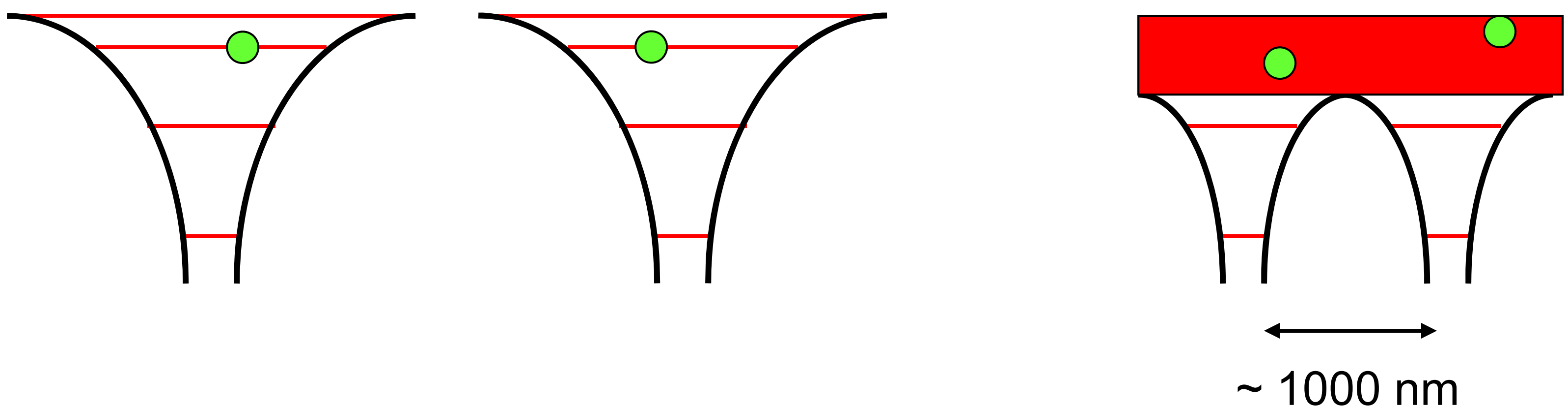
Atoms within bonding distance



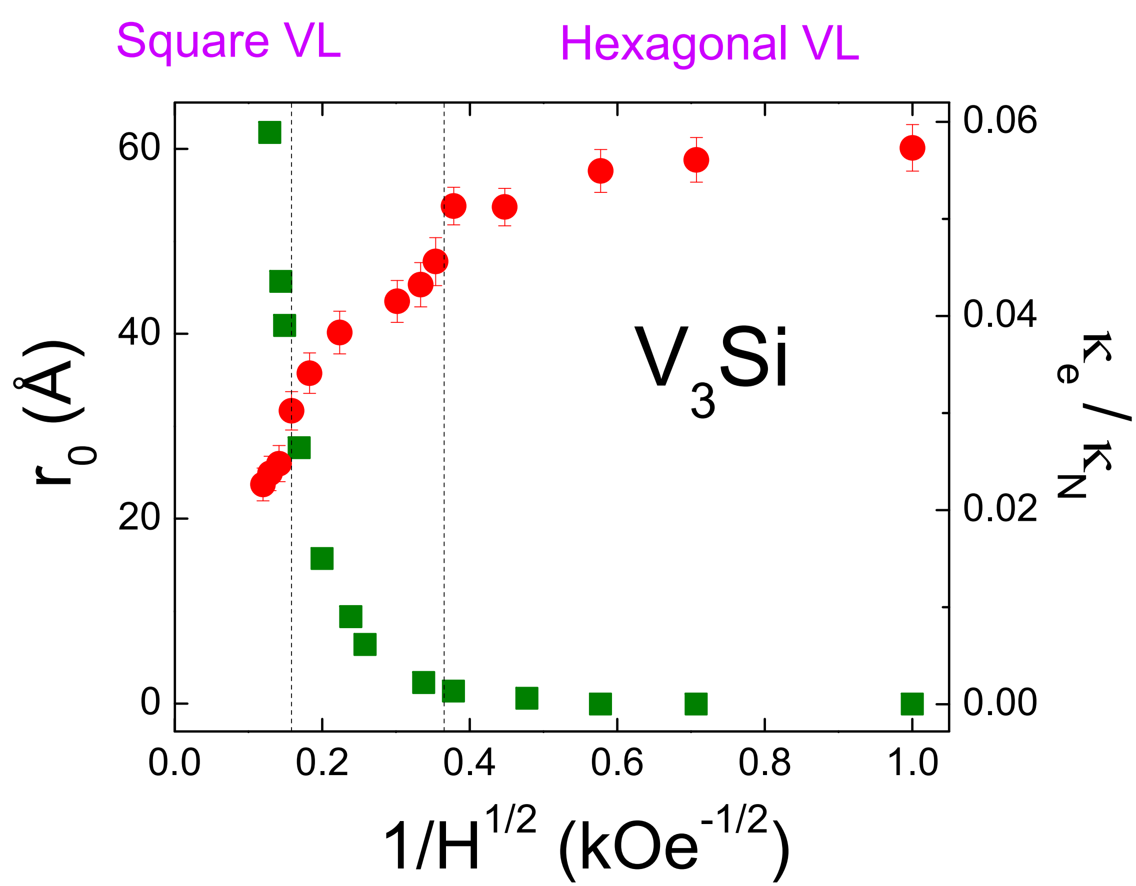
Intervortex Quasiparticle Transfer in a Superconductor

Vortices far apart

Vortices strongly overlap



The magnetic field dependence of the ***vortex core size*** r_0 in V_3Si measured by μSR can be compared to electronic **thermal conductivity measurements**, which are directly sensitive to delocalized quasiparticles.

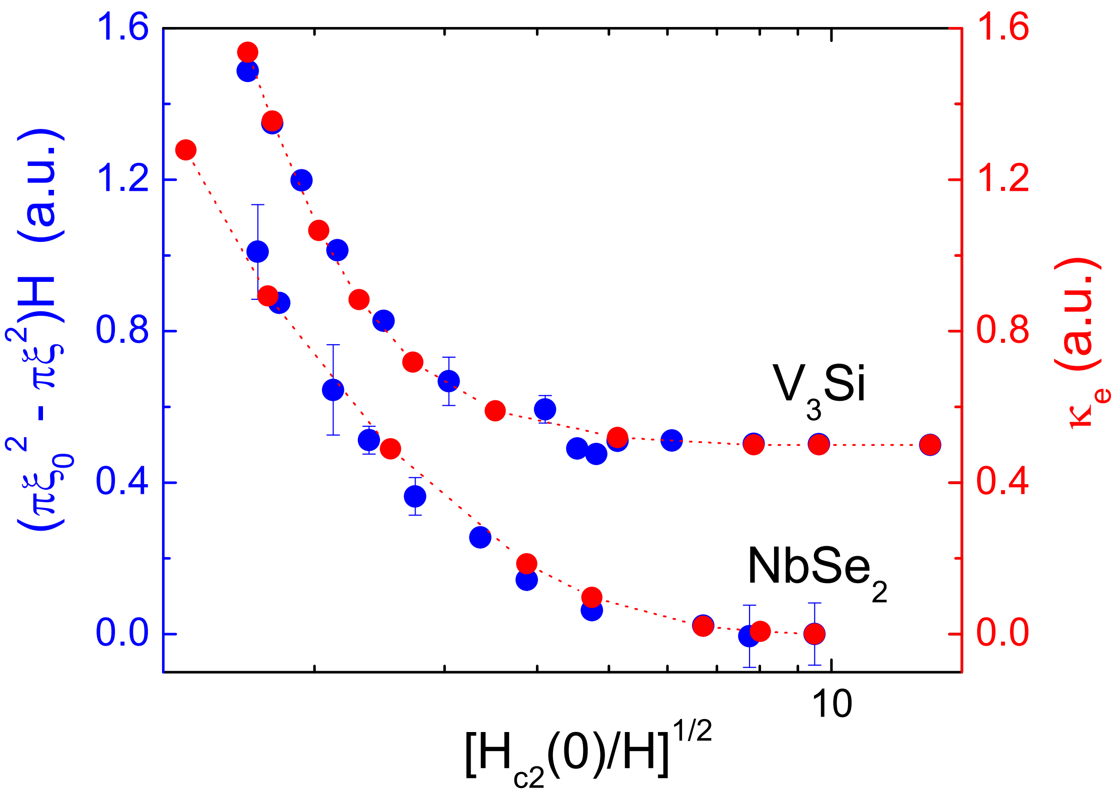


Shrinking vortex cores are attributed to the ***delocalization*** of bound quasiparticle core states.

μSR & electronic thermal conductivity

The vortex cores contain **localized** or bound quasiparticle states, with a density proportional to the product of the vortex core area (~ $\pi\xi^2$) and the applied magnetic field *H*. Since the electronic thermal conductivity is a measure of **delocalized** quasiparticles only, the following relation is expected to hold:

$$\kappa_e \sim N(0)_{deloc} = N(0)_{tot} - N(0)_{loc} \sim [\pi\xi(H_{c1})^2 - \pi\xi(H)^2]\, H$$

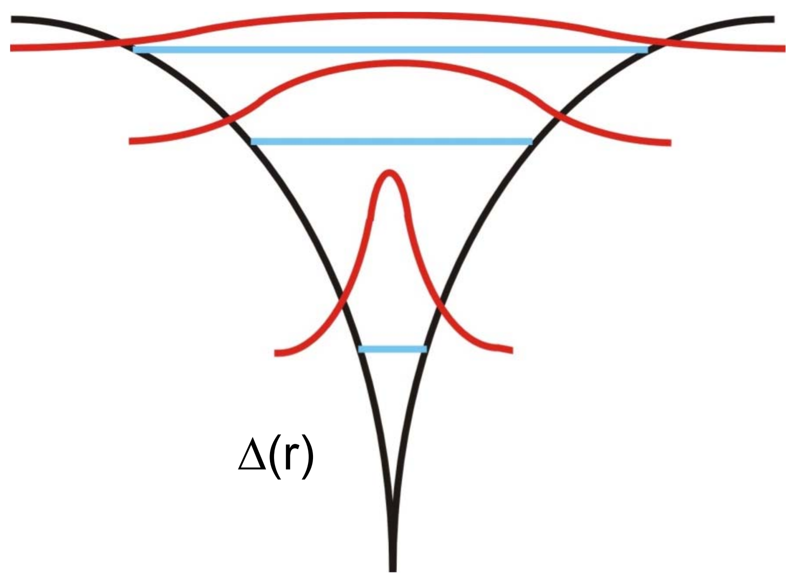


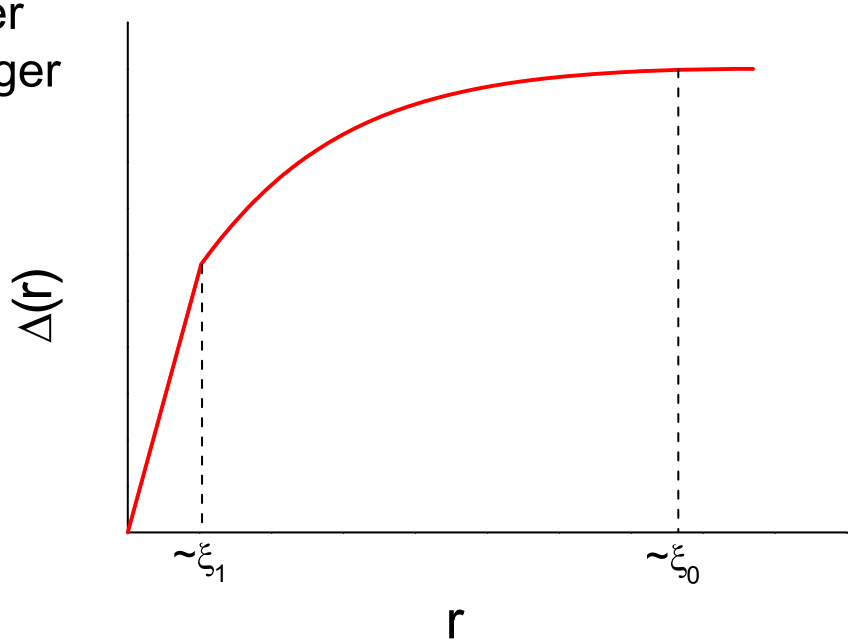
JES, Rep. Prog. Phys. **70**, 1717-1755 (2007)

Kramer-Pesch Effect

Thermal population of higher energy core states leads to an ***expansion of the core radius*** with increasing T, because the higher energy bound states extend outwards to larger radii.

L. Kramer & W. Pesch, Z. Phys. **269**, 59 (1974)

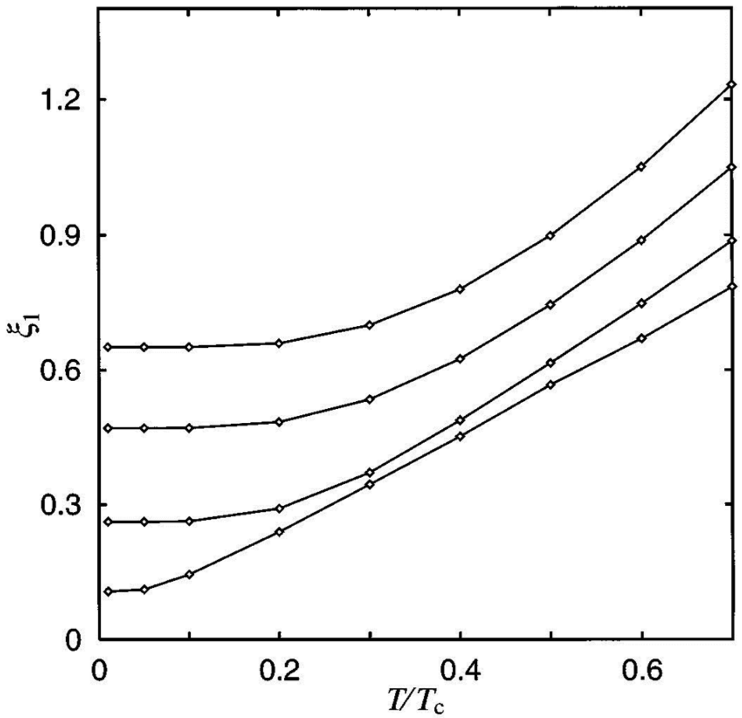




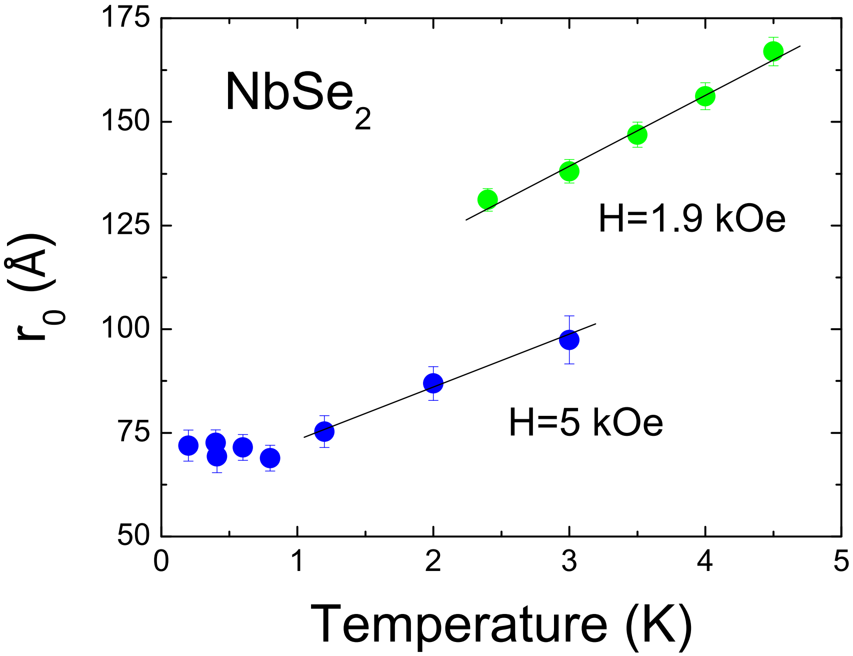
$$\xi_1 \approx \xi_0 \; T/T_c$$

ξ_1 is slope of $\Delta(r)$ at vortex core center

Kramer-Pesch effect observed by μSR



Hayashi *et al.*, PRL **80**, 2921 (1998)

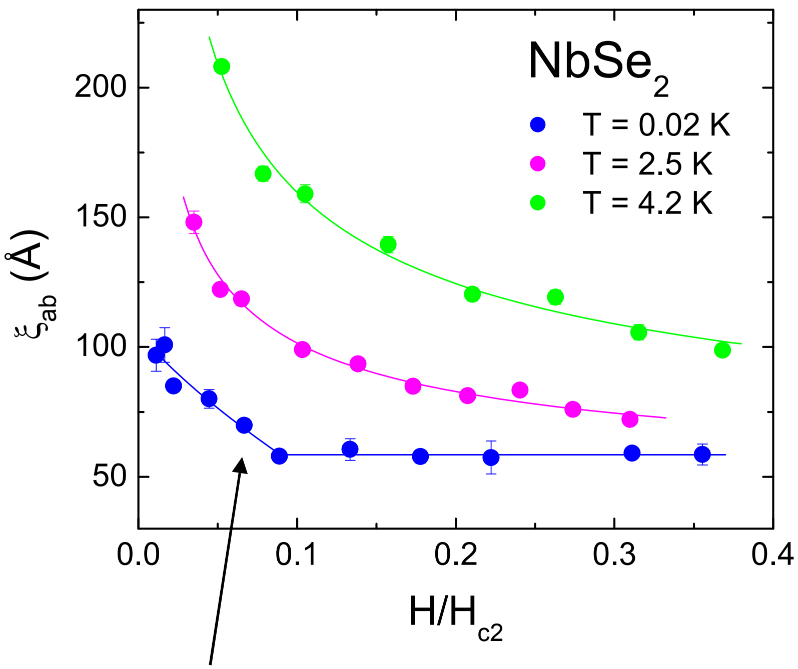


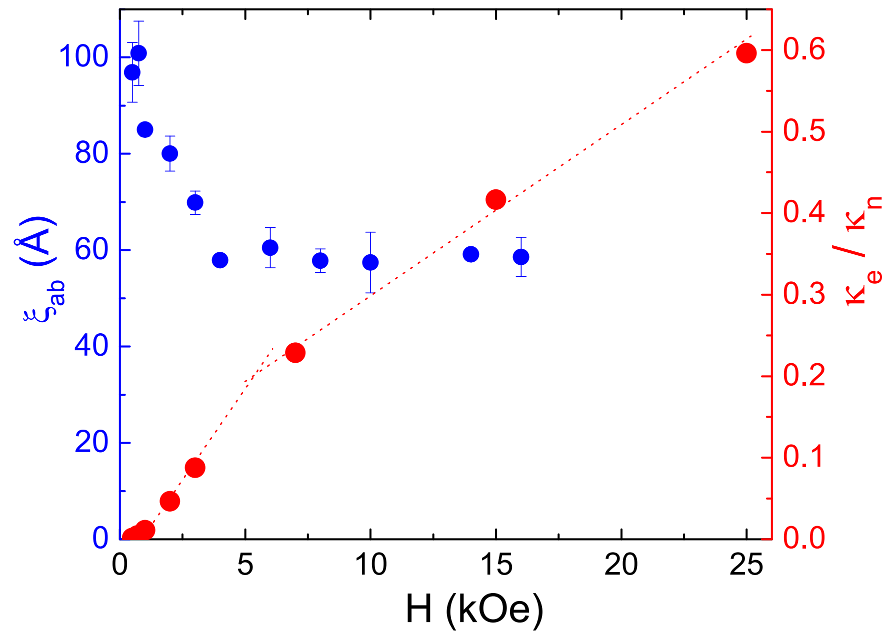
J.E. Sonier *et al.*, PRL **79**, 1742 (1997)
R.I. Miller *et al.*, PRL **85**, 1540 (2000)

Vortex core size in the multiband superconductor $NbSe_2$

$NbSe_2$ has **two** distinct superconducting energy gaps (*large* and *small*) on different Fermi sheets.

At low fields the ***loosely bound*** quasiparticle states associated with the **smaller** energy gap dominate the vortex structure, but delocalize at higher magnetic field where these states strongly overlap.





Freeze out thermal excitations of quasiparticle core states to reveal *multiband vortices*.

F.D. Callaghan *et al.*, Rep. Phys. Rev. Lett. **95**, 197001 (2005)