Relaxonomy

A Chrestomathy of Muon Spin Relaxation (and "Relaxation") Functions

"borrowed" from various sources by

Jess H. Brewer

UBC Physics & Astronomy

chrestomathy
1. a collection of literary selections, especially in a foreign language, as an aid to learning.

relaxonomy
0. Not a dictionary word. (*yet*)

Brewer's List of μSR Acronyms

Transverse Field

Longitudinal Field

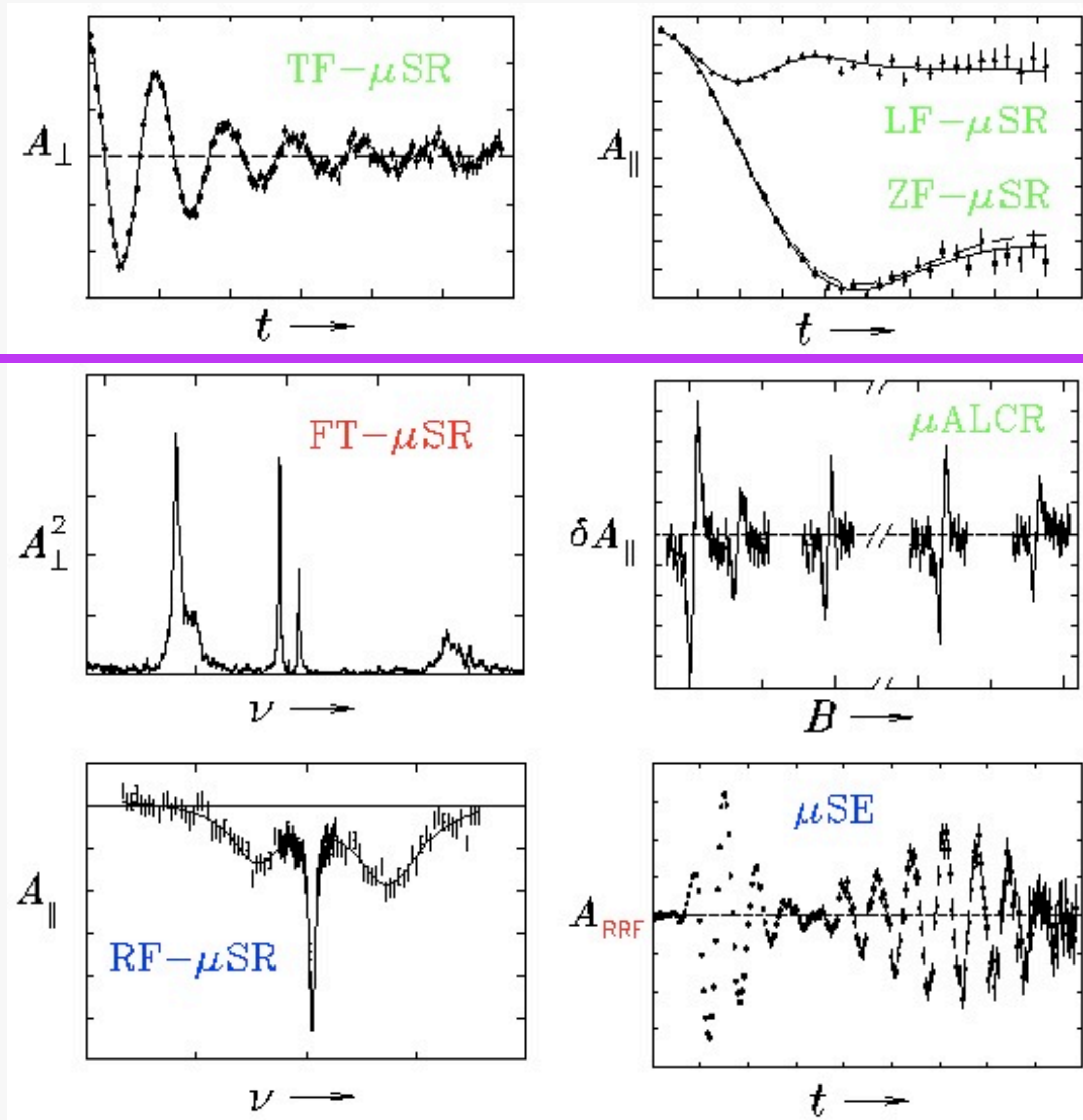
Zero Field

Fourier Transform μSR

Avoided Level Crossing Resonance

Muon Spin Resonance

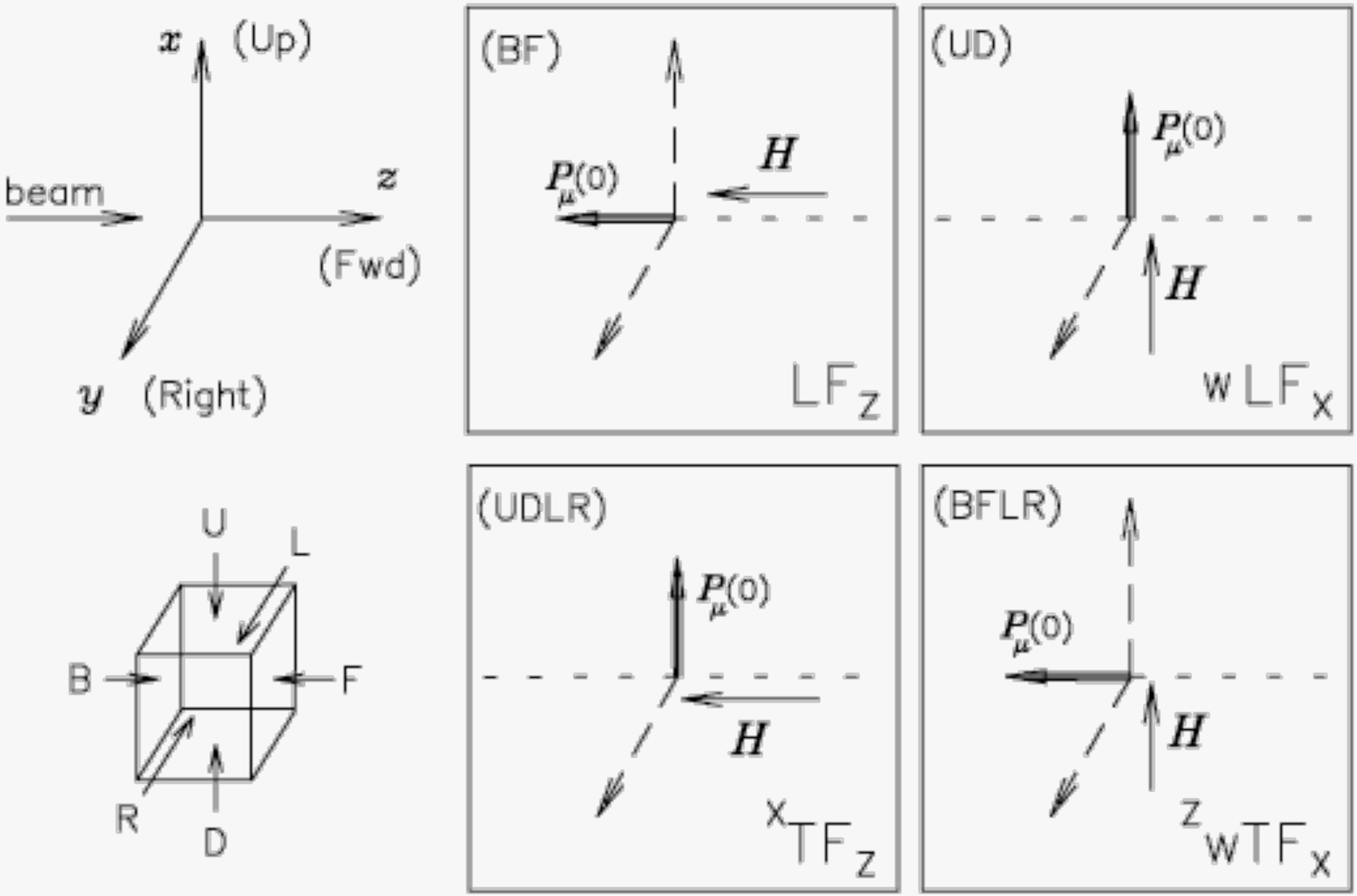
Muon Spin Echo



Wednesday, August 10, 2011

TRIUMF

Coordinate Conventions



TENTATIVE OUTLINE

- $G_{ij}(t) = \langle P_i(0)\, P_j(t) \rangle$ formalism
- Basic TF $G_{TF}(t)$ forms
  - Exponentials & Gaussians
  - Abragam's dynamic $G_{TF}(t,\sigma,\tau)$
- "Static" ZF/LF forms
  - Kubo-Toyabe functions
    - Gaussian $g_{zz}(t,\Delta_G, H_z)$
    - Lorentzian $g_{zz}(t, \Delta_L, H_z)$
    - Generalized $g_{zz}(t, \Delta)$ [deferred]
  - QM correct forms
    - "FμF" in fluorides
    - Copper (O-site)
- "Dynamicization" methods
  - Strong Collision model
  - Spin Glass dynamics
- "Stretched Exponentials"
  - Legitimate Uses
  - Bogus Empiricism
- Double Relaxation
- Exotic Forms
  - Distributions of Exponentials
  - Arrival-Time Distributions
  - More Bogus Empiricism

Time Correlation Functions

$G_{ij}(t) = \langle P_i(0)\, P_j(\mathrm{t}) \rangle$ is the average correlation between the initial muon polarization component in the i direction and the component in the j direction at time t.

Thus $G_{zz}(t) = \langle P_z(0)\, P_z(\mathrm{t}) \rangle$ is the "relaxation function" for the polarization component parallel to the applied field $\boldsymbol{B}$ (along z). $G_{zz}(t)$ is also known as the "longitudinal", "ZF" or "LF" relaxation function, $G_{\mathrm{LF}}(t)$. This is valid only as long as $\boldsymbol{P}(0)$ is along z and there are no local fields $\perp$ to z; otherwise transverse components may develop.

The situation is more complicated with the "transverse" or "TF" correlation functions, which will be oscillatory due to precession of $\boldsymbol{P}$ about $\boldsymbol{B}$:

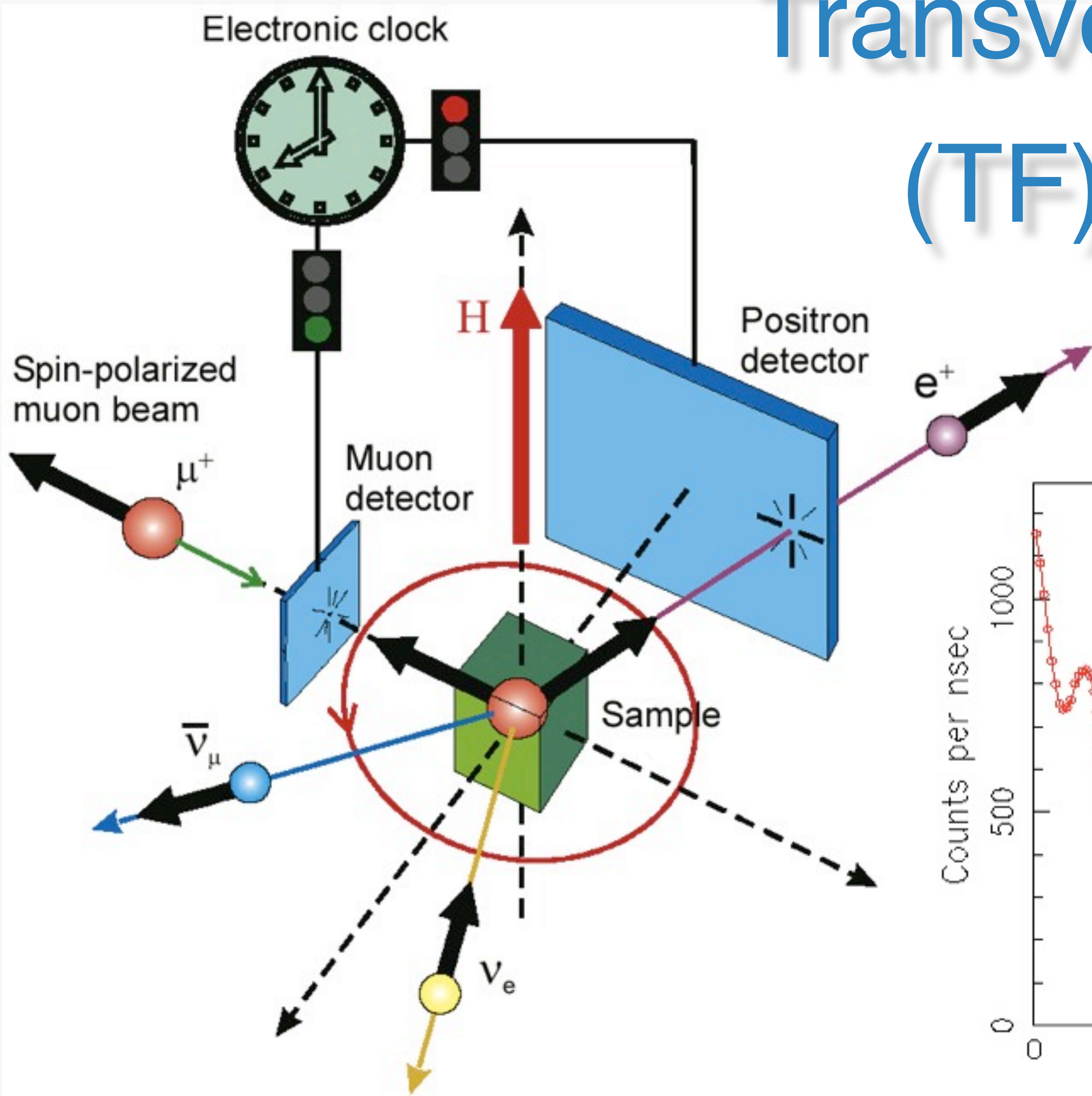
$$G_{xx}(t) = \langle P_x(0)\, P_x(\mathrm{t}) \rangle = G_{\mathrm{TF}}(t) \cos(\gamma_\mu B t + \varphi)$$

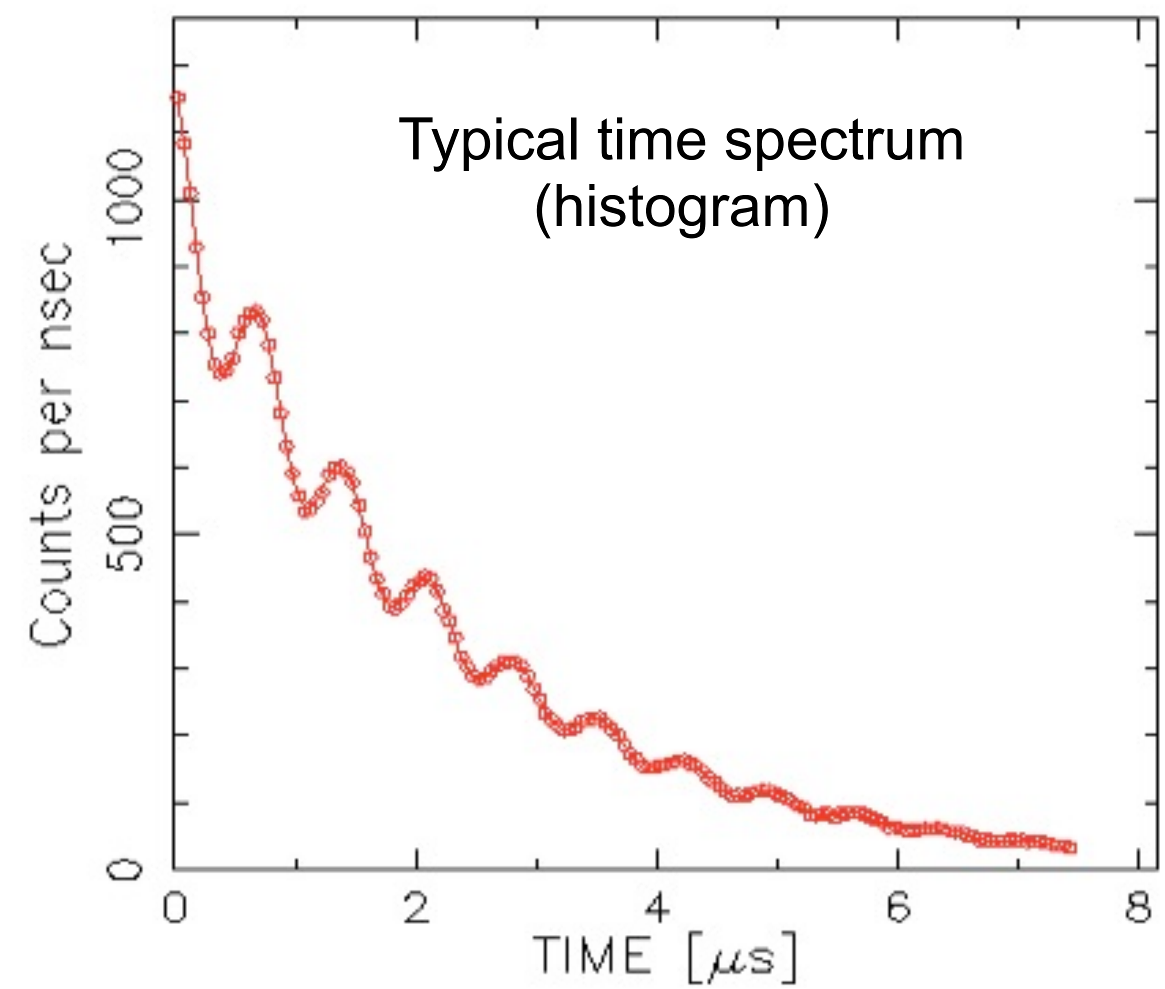
and

$$G_{yy}(t) = \langle P_y(0)\, P_y(\mathrm{t}) \rangle = G_{\mathrm{TF}}(t) \sin(\gamma_\mu B t + \varphi)$$

where φ is the angle between $\boldsymbol{P}(0)$ & $\boldsymbol{x}$ and $G_{\mathrm{TF}}(t)$ is the "TF relaxation function", usually assumed to be a simple, real "envelope" function, but sometimes it is not. Sadly, it is customary to write $G_{xx}(t)$ or $G_x(t)$ to mean $G_{\mathrm{TF}}(t)$. So I will, sometimes.

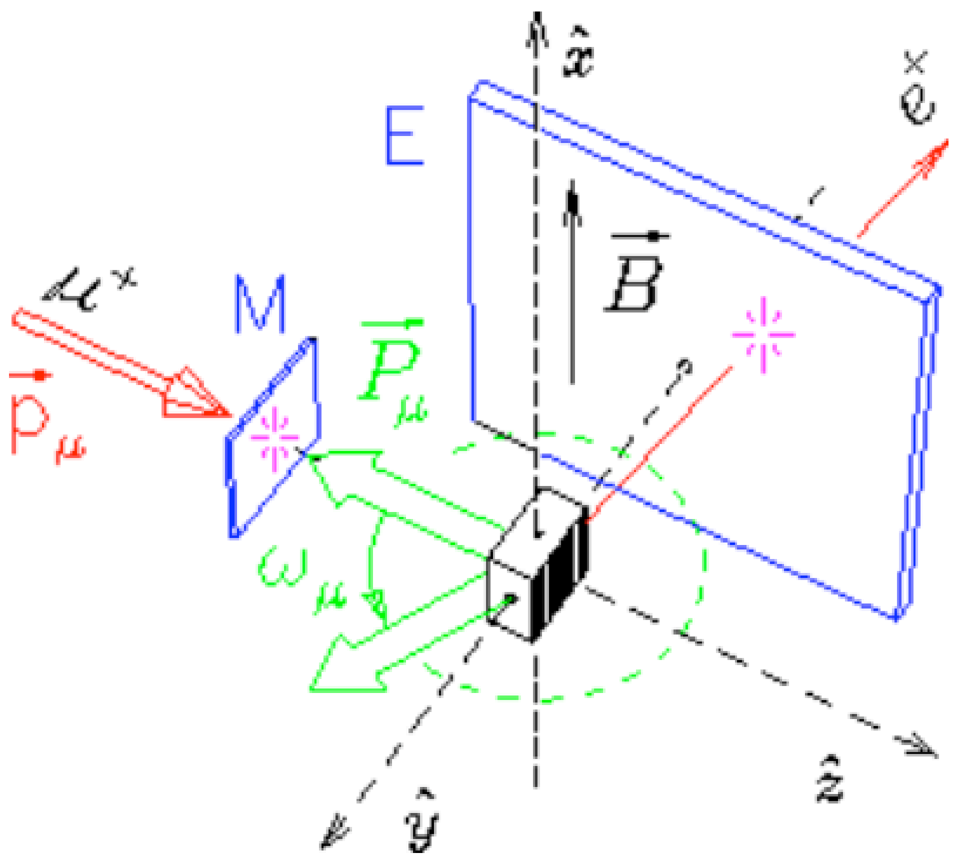
Transverse Field (TF)-μ^+SR

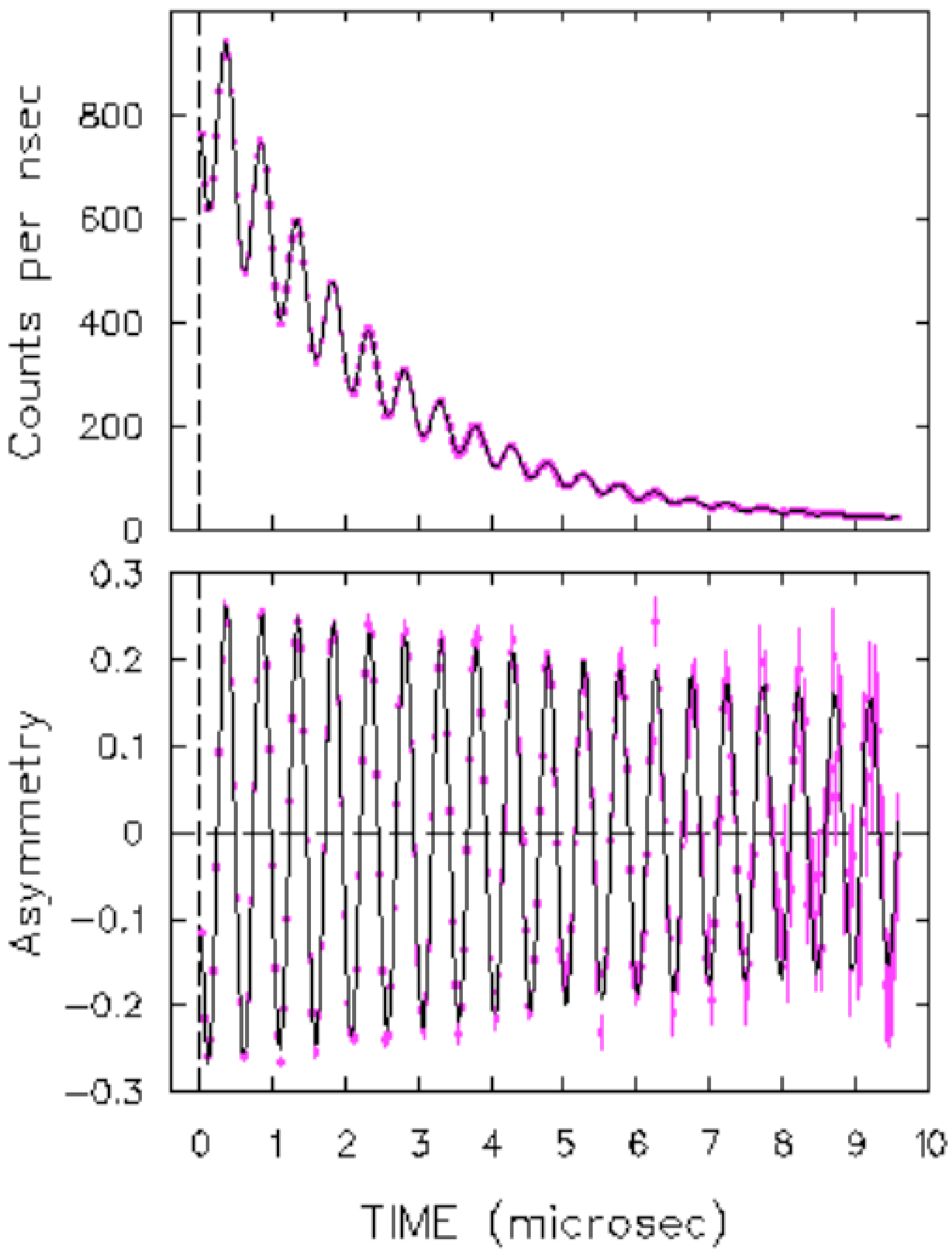




wTF-μ⁺SR:

$$N(t) = N_0 \left\{ B + e^{-t/\tau_\mu} \left[1 + A_0 G_{xx}(t) \cos(\omega_\mu t + \phi) \right] \right\}$$





$$A(t) = [N(t) - N_0 B]\, e^{+t/\tau_\mu} - 1$$

$$= A_0 G_{xx}(t) \cos(\omega_\mu t + \phi)$$

Single-Histogram Asymmetry

Referring to formula for *N*(*t*), **if we know N_0 and *B*** we can extract the asymmetry *A*(*t*).

$$A(t) = \left[\frac{N(t) - B}{N_0}\right] e^{+t/\tau_\mu} - 1$$

How can we *determine* N_0 and *B* *numerically* from a single histogram? *As long as any oscillations are fast enough to be averaged over times short compared to the muon lifetime*, we only need to make the following sums over the "raw" time spectrum from start (at t_i) to finish (at t_f):

$$N_0 = \frac{S\tau_\mu E_+ - RT}{\tau_\mu^2 E_+ E_- - T^2}$$

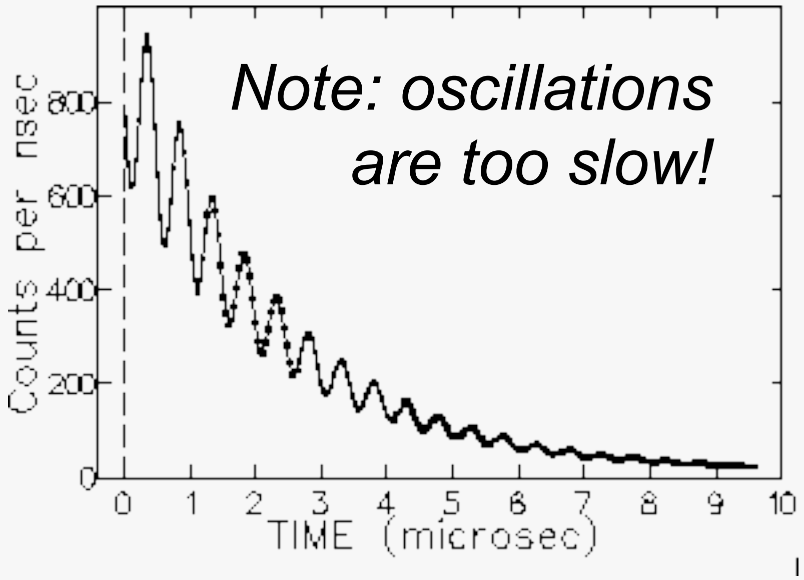
$$B = \frac{R\tau_\mu E_- - ST}{\tau_\mu^2 E_+ E_- - T^2}$$

$$S \equiv \int_{t_i}^{t_f} N(t)dt$$

$$R \equiv \int_{t_i}^{t_f} N(t)\exp(t/\tau_\mu)dt$$

$$E_\pm \equiv \pm\left[\exp(\pm t_f/\tau_\mu) - \exp(\pm t_i/\tau_\mu)\right]$$

$$T \equiv t_f - t_i$$

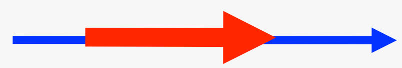


Note: oscillations are too slow!

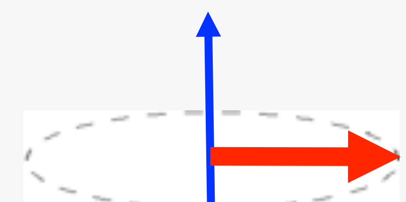
TRIUMF

Motion of Muon Spins in Static Local Fields


$\boldsymbol{S}_\mu$ = Expectation value of a muon's spin direction

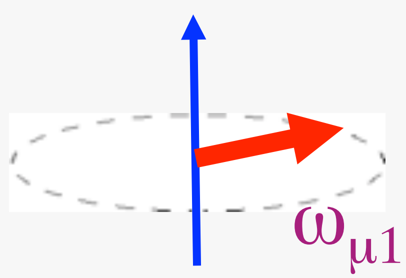
(a) All muons "see" same field $\boldsymbol{B}$:  for $\boldsymbol{B} \parallel \boldsymbol{S}_\mu$ nothing happens.

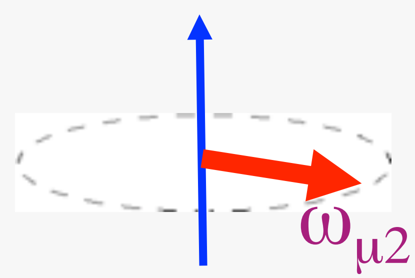
$\omega_\mu = \gamma_\mu |\boldsymbol{B}|$ for $\boldsymbol{B} \perp \boldsymbol{S}_\mu$ Larmor precession:



$\gamma_\mu / 2\pi = 135.5$ MHz/T

(b) All muons "see" $|\boldsymbol{B}_0| \pm$ random $\Delta\boldsymbol{B}$ in the **same direction**:





Different $\boldsymbol{S}_\mu$ **precess** at different ω_μ

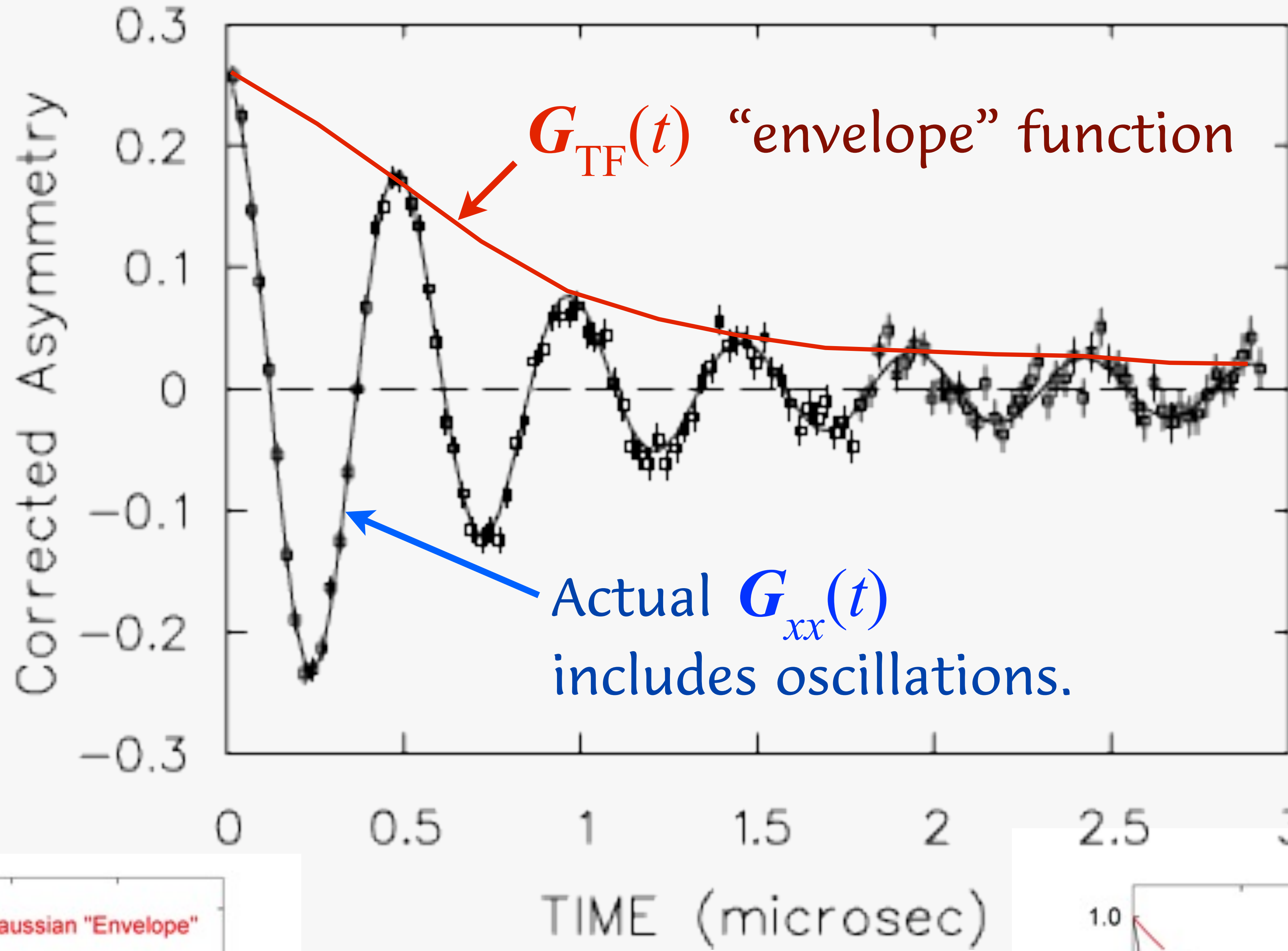
$\Rightarrow$ get out of phase ("**dephasing**").

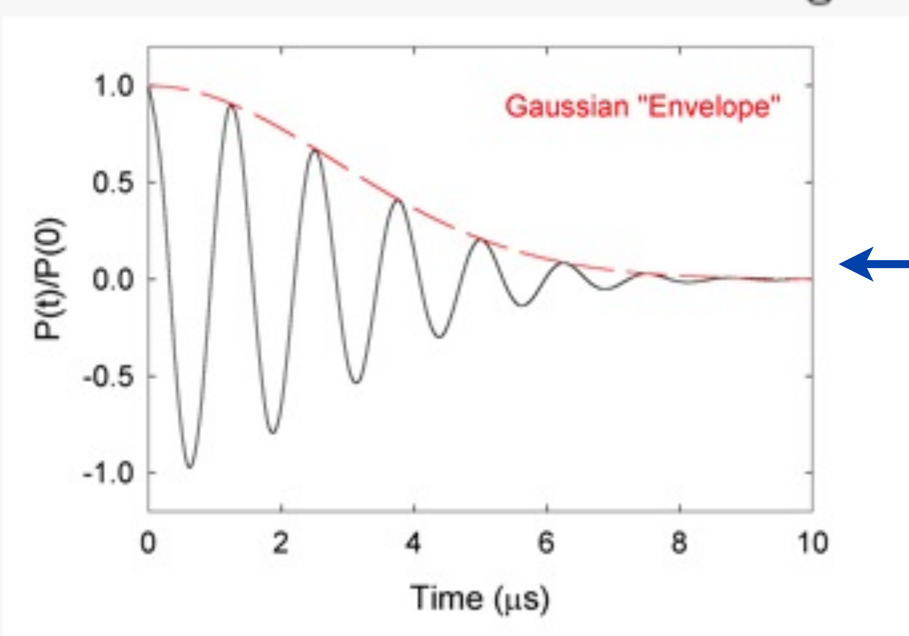
(c) But $\boldsymbol{S}_\mu$'s can "refocus" after a "π pulse" of RF!

$\Rightarrow$ $\boldsymbol{S}_\mu$ "**order**" is **not lost**, just **hidden**. **Dephasing** is **not true relaxation**!

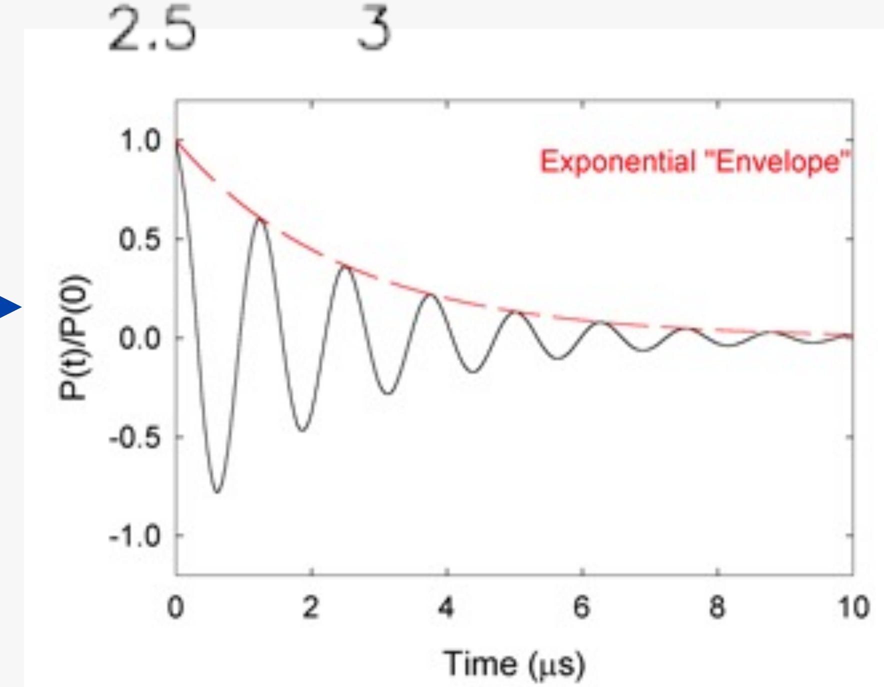
TRIUMF

TF "Relaxation"





Familiar shapes:
what determines
which we will see?

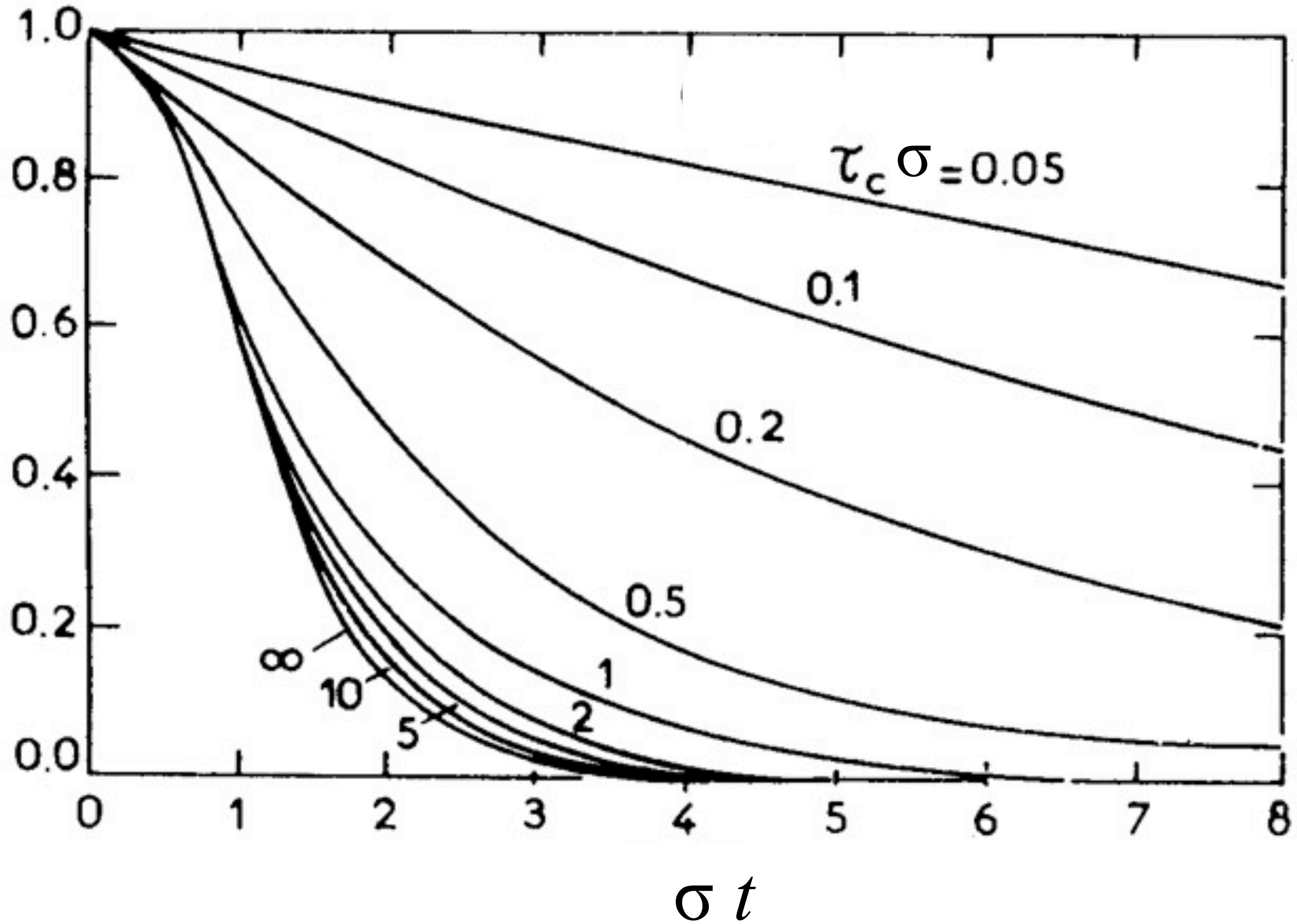


Fluctuations in TF:

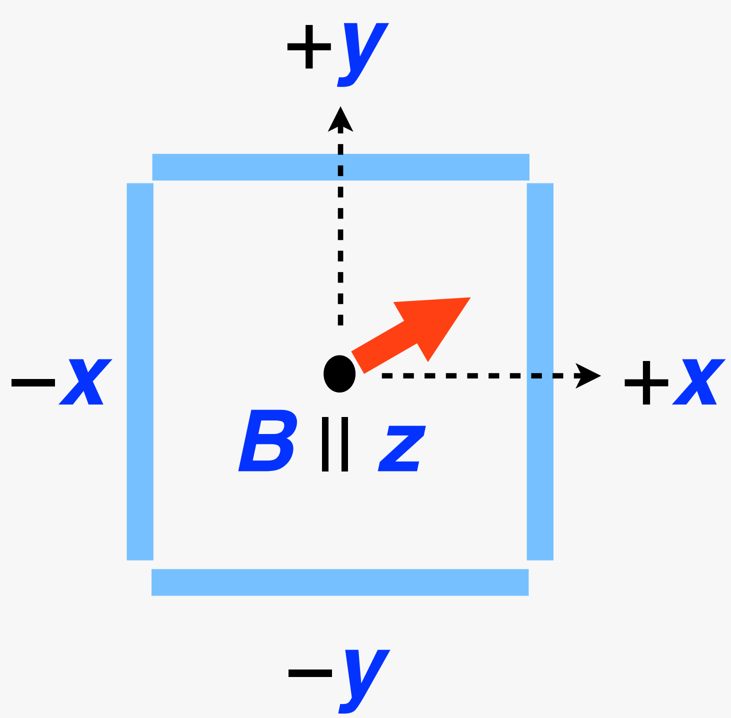
Abragam's *envelope function*: $G_{TF}(t) = e^{-\Theta}$

where $\Theta = (\sigma^2/\nu_c^2)\{e^{-\nu_c t} - 1 + \nu_c t\}$,

σ = static Gaussian width, $\nu_c = 1/\tau_c$ = fluctuation rate



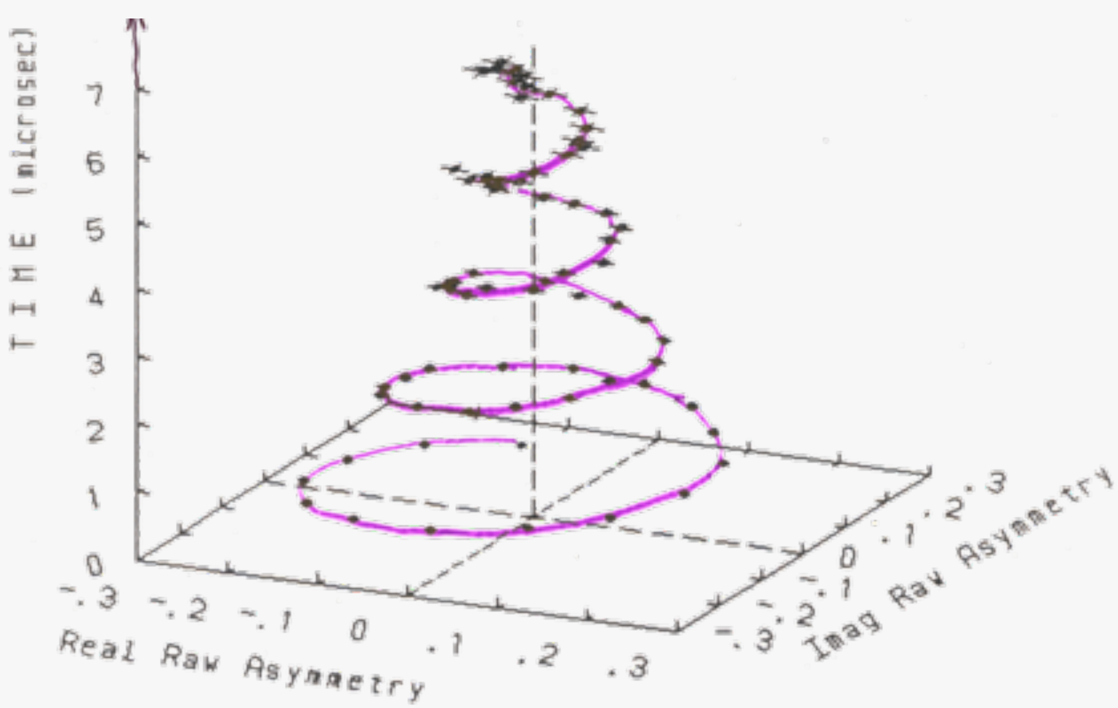
Complex TF Asymmetry



$$\mathcal{Re}\,\mathcal{A}(t) = A_x(t) = \left[A_x^+(t) - A_x^-(t)\right]/2$$

$$\mathcal{Im}\,\mathcal{A}(t) = A_y(t) = \left[A_y^+(t) - A_y^-(t)\right]/2$$

$$\mathcal{A}(t) = A_x(t) + i\,A_y(t)$$

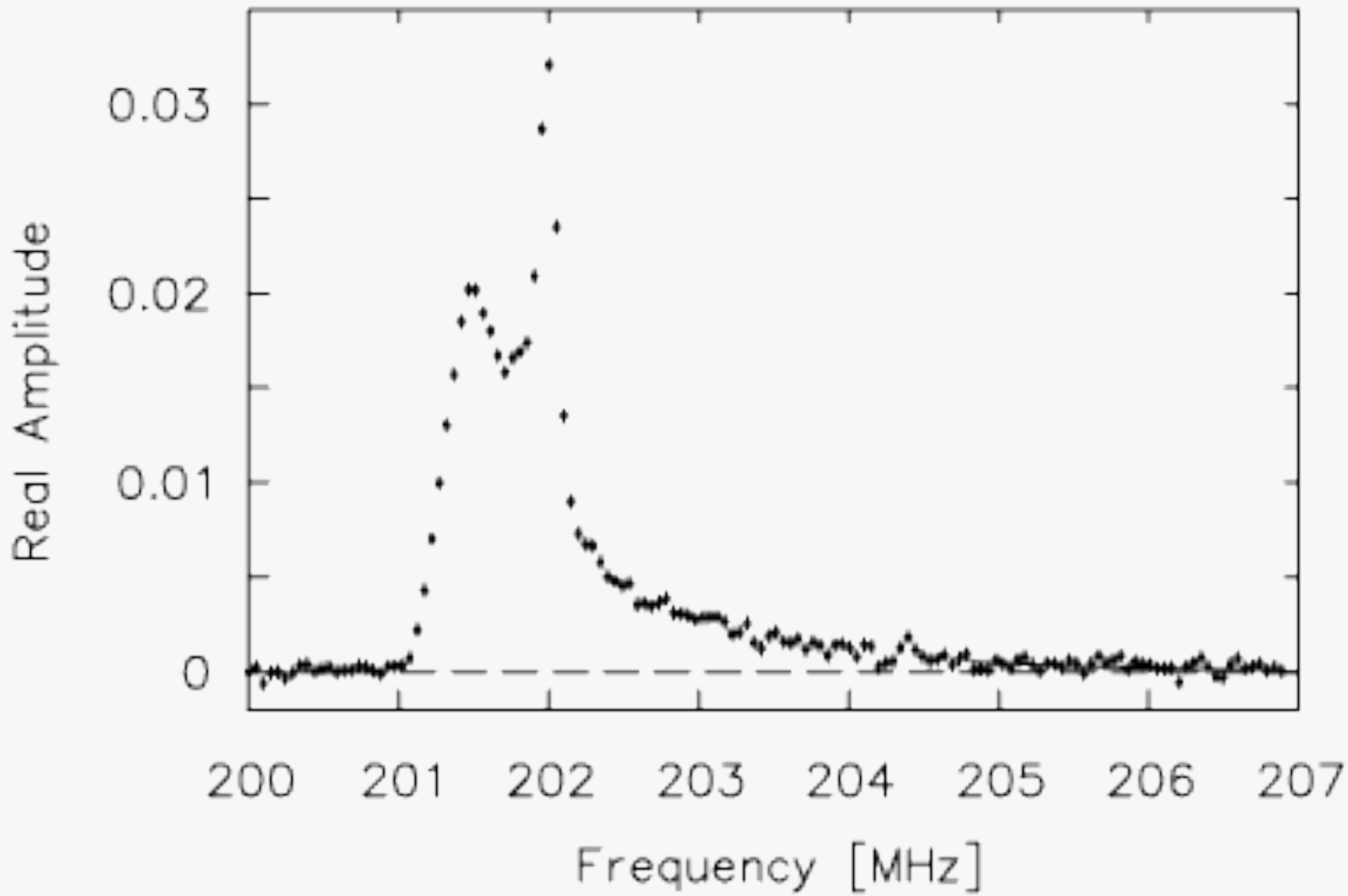


For High Transverse Field (HTF), transform into the Rotating Reference Frame (RRF):

$$\mathcal{A}_{\mathrm{LAB}}(t) = A_x(t) + iA_y(t)$$

$$\mathcal{A}_{\mathrm{RRF}}(\Omega, t) = e^{-i\Omega t}\,\mathcal{A}_{\mathrm{LAB}}(t)$$

Anisotropic Lineshapes



$G_{xx}(t)$ includes oscillations.

$G_{TF}(t)$ "envelope" function is complex (also includes oscillations).

TRIUMF

Motion of Muon Spins in Static Local Fields

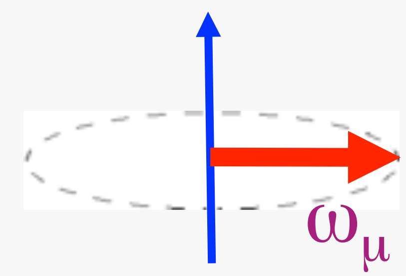
$\vec{S}_\mu$ = Expectation value of a muon's spin direction

(a) All muons "see" same field $\boldsymbol{B}$: for $\boldsymbol{B} \parallel \boldsymbol{S}_\mu$ nothing happens.

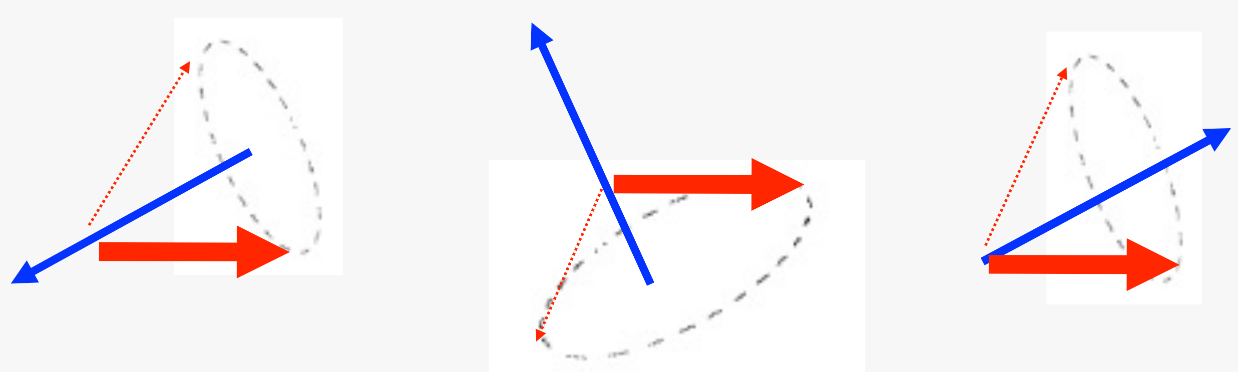
$\omega_\mu = \gamma_\mu |\boldsymbol{B}|$

$\gamma_\mu / 2\pi = 135.5$ MHz/T

for $\boldsymbol{B} \perp \boldsymbol{S}_\mu$ Larmor precession:



(b) All muons "see" same $|\boldsymbol{B}|$ but **random direction** :



2/3 of $\boldsymbol{S}_\mu$ **precesses** at ω_μ

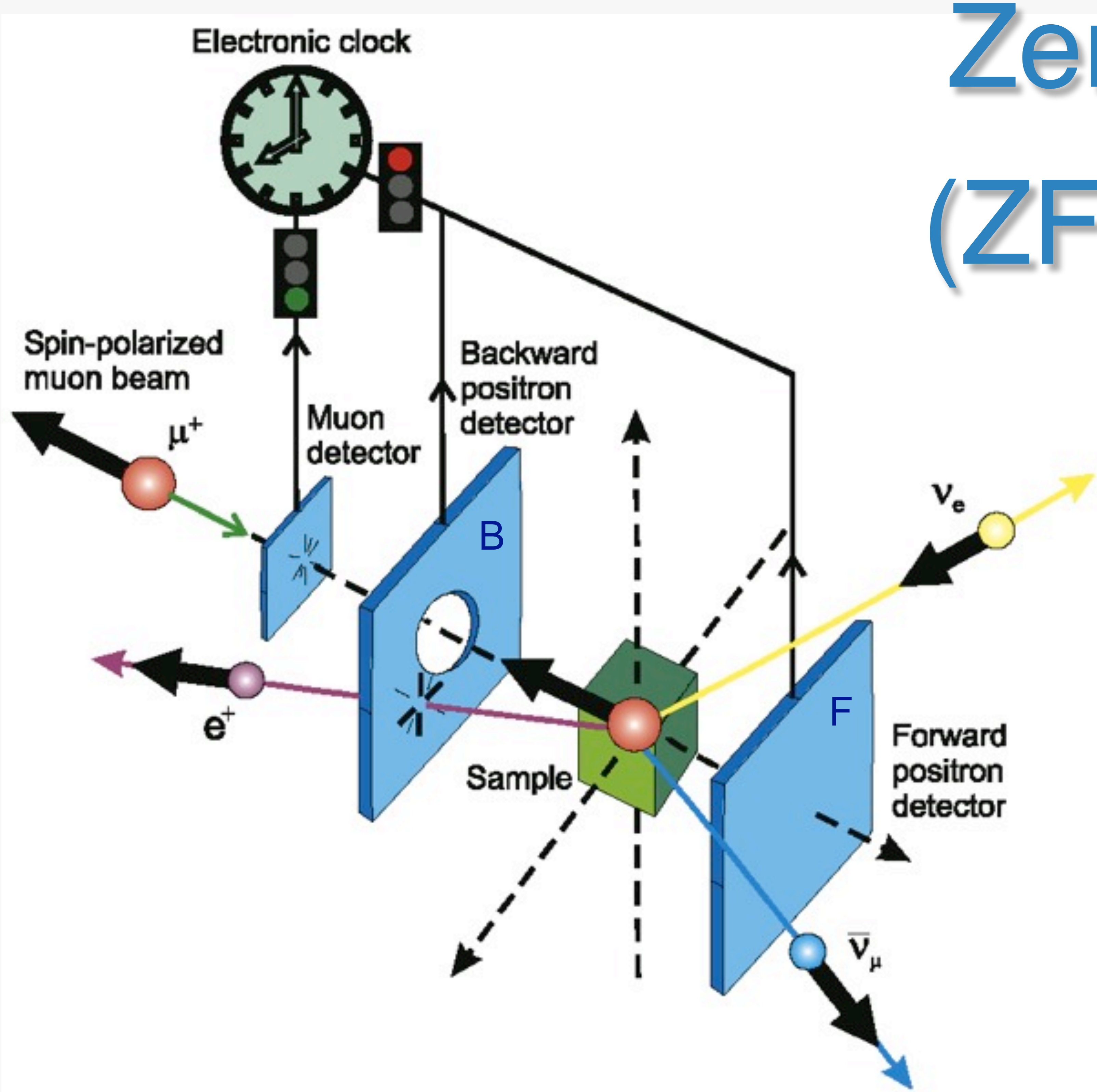
1/3 of $\boldsymbol{S}_\mu$ **stays constant**

(c) Local field $\boldsymbol{B}$ **random** in **both** magnitude **and** direction:

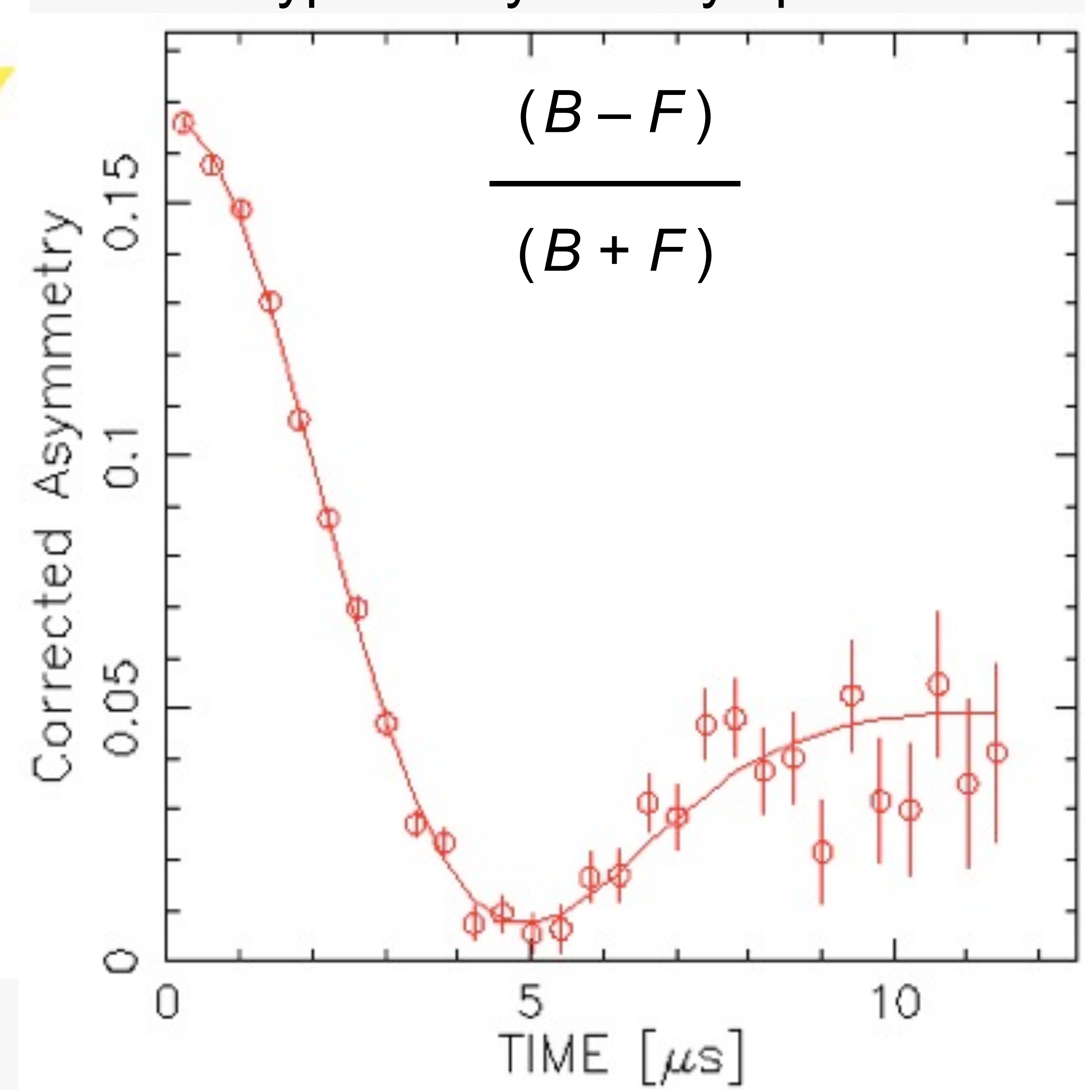
All ↗ do not return to the same orientation at the same time

(dephasing) $\Rightarrow$ $\boldsymbol{S}_\mu$ "**relaxes**" as $G_{zz}(t)$ [Kubo & Toyabe, 1960's]

Zero Field (ZF)-μ^+SR



Typical asymmetry spectrum



Two-Histogram Asymmetry

How do we formally extract the asymmetry $A(t)$ from the histograms of two opposing detectors, *e.g.* "U" (up) and "D" (down), if the time dependence of $A(t)$ is *slow* relative to the muon lifetime?

$$N_{\mathrm{U,D}}(t) = B_{\mathrm{U,D}} + N_0 \epsilon_{\mathrm{U,D}} [1 \pm A_{\mathrm{U,D}} P_x(t)]$$

The "raw" asymmetry $a(t)$ is formed as the *ratio of the difference to the sum* of the "background-subtracted" raw histograms.

$$a(t) \equiv \frac{[N_U(t) - B_U] - [N_D(t) - B_D]}{[N_U(t) - B_U] + [N_D(t) - B_D]}$$

The "corrected" asymmetry $A_U P_x(t)$ is formed using the empirical ratios of the positron detection efficiencies and intrinsic asymmetries of the two detectors. (These ratios are usually determined by fitting wTF spectra in the same apparatus.)

$$A_U P_x(t) = \frac{(\alpha - 1) + (\alpha + 1) a(t)}{(\alpha\beta + 1) + (\alpha\beta - 1) a(t)}$$

$$\alpha \equiv \frac{\epsilon_D}{\epsilon_U} \qquad \beta \equiv \frac{A_D}{A_U}$$

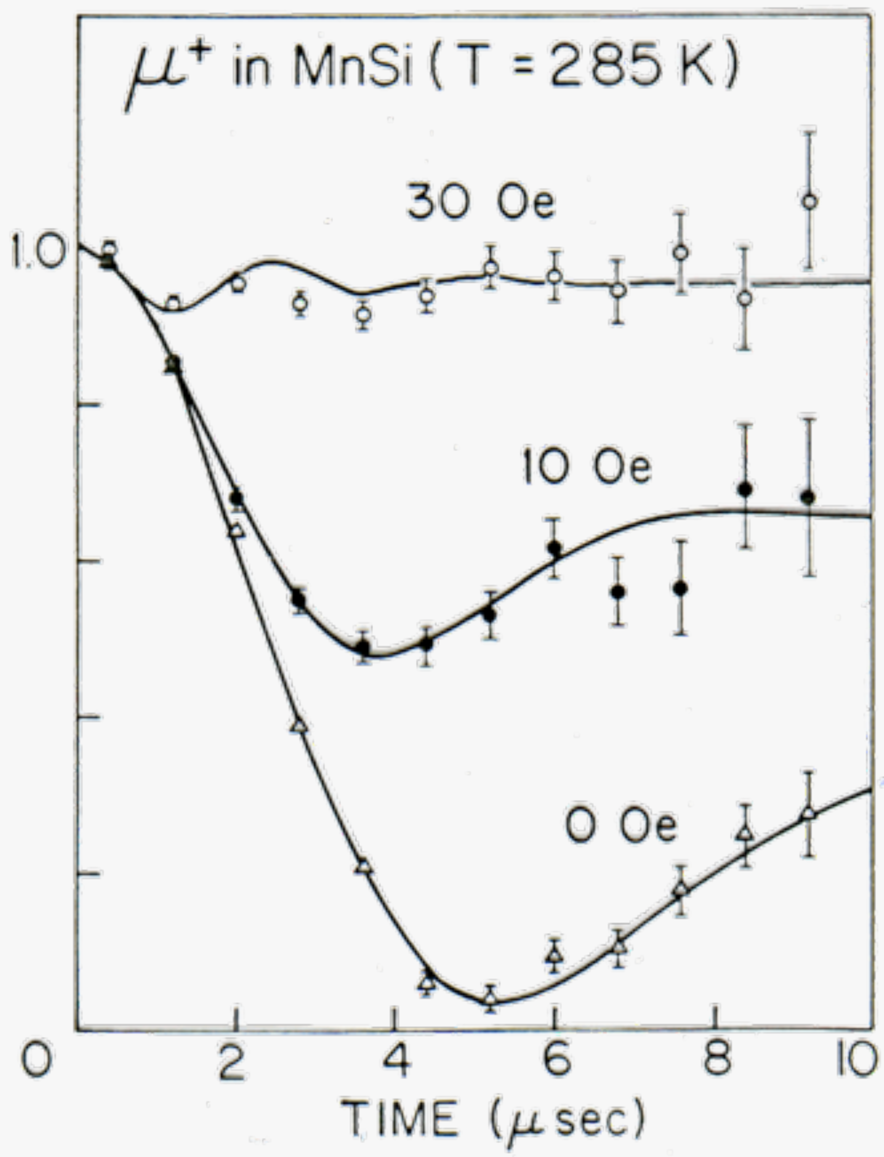
Note: "baseline" [for $P_x(t) = 0$] is $(1-\alpha)/(1+\alpha)$

Static Gaussian Kubo-Toyabe functions

$$g_{KT}^{GLF}(\Delta_G, t, \omega_L) = 1 - \frac{2\Delta_G^2}{\omega_L^2}\left\{1 - \cos(\omega_L\, t)\ \exp\left[-\frac{(\Delta_G\, t)^2}{2}\right]\right\} + \frac{2\Delta_G^4}{\omega_L^3}\int_0^t \sin(\omega_L\, t)\ \exp\left[-\frac{(\Delta_G\, \tau)^2}{2}\right]\ d\tau$$

$$(\omega_L = \gamma_\mu H)$$

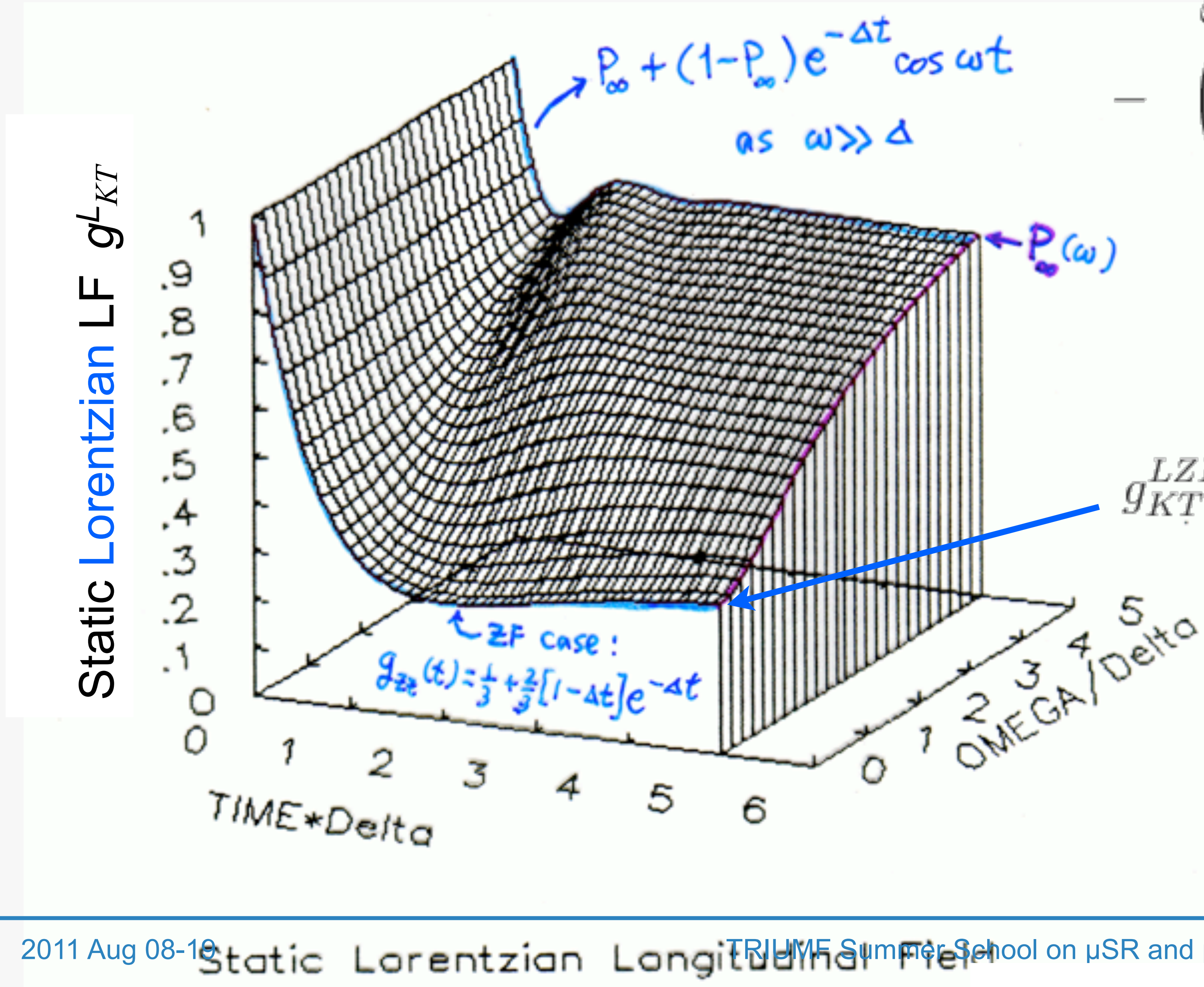
Static Gaussian LF $g^G{}_{KT}$



$$g_{KT}^{GZF}(\Delta_G, t) = \frac{1}{3} + \frac{2}{3}\cdot\left[1 - (\Delta_G\, t)^2\right]\cdot\exp\left[-\frac{(\Delta_G\, t)^2}{2}\right]$$

Static Lorentzian Kubo-Toyabe functions

$$
\begin{aligned}
g_{KT}^{LLF}(\Delta_L, t, \omega_L) = 1 \;&-\; \frac{\Delta_L}{\omega_L} j_1(\omega_L\, t)\, e^{-\Delta_L\, t} \\
&-\; \frac{\Delta_L^2}{\omega_L^2}\left[j_0(\omega_L\, t)\, e^{-\Delta_L\, t} - 1\right] \\
&-\; \left(1 + \frac{\Delta_L^2}{\omega_L^2}\right)\Delta_L \int_0^t j_0(\omega_L\, \tau)\, e^{-\Delta_L\, \tau}\, d\tau
\end{aligned}
$$



$$
g_{KT}^{LZF}(\Delta_L, t) = \frac{1}{3} + \frac{2}{3}\cdot(1 - \Delta_L\, t)\cdot e^{-\Delta_L\, t}
$$

1983-1985: μSR discovers Quantum Mechanics

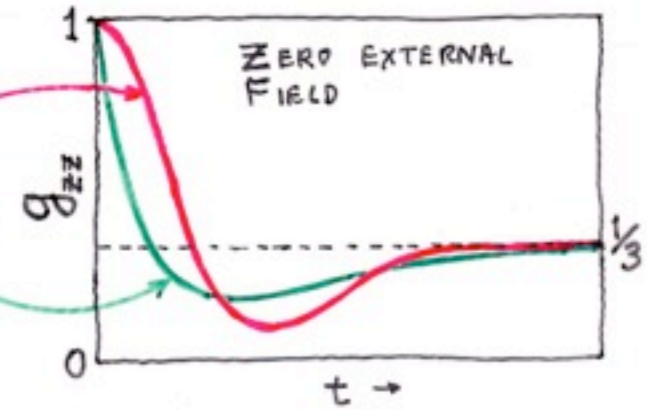
- BEFORE 1983: Classical Kubo-Toyabe theory of muon relaxation: Each μ^+ "sees" STATIC $\vec{B}_{loc}$ from nearby dipoles:

For isotropic $\mathcal{D}(\vec{B}_{loc})$, $[\vec{\omega}_\mu = \gamma_\mu \vec{B}_{loc}]$

$g_{zz}(t) \equiv \langle P_z^\mu(t) \rangle = \frac{1}{3} + \frac{2}{3} \langle \cos \omega_\mu t \rangle$

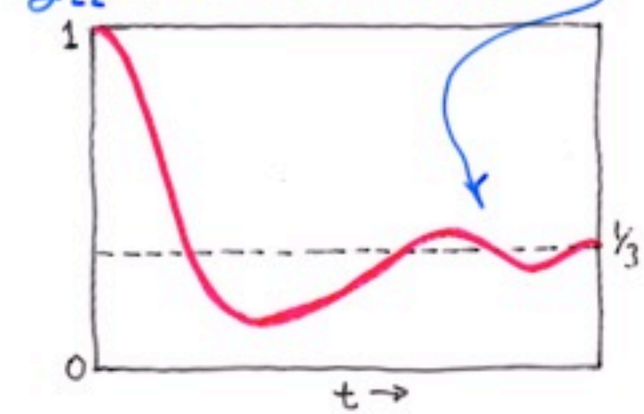
Regular lattice of dipoles
⇒ Gaussian $\mathcal{D}(\vec{B}_{loc})$

Random, dilute large moments
⇒ Lorentzian $\mathcal{D}(\vec{B}_{loc})$



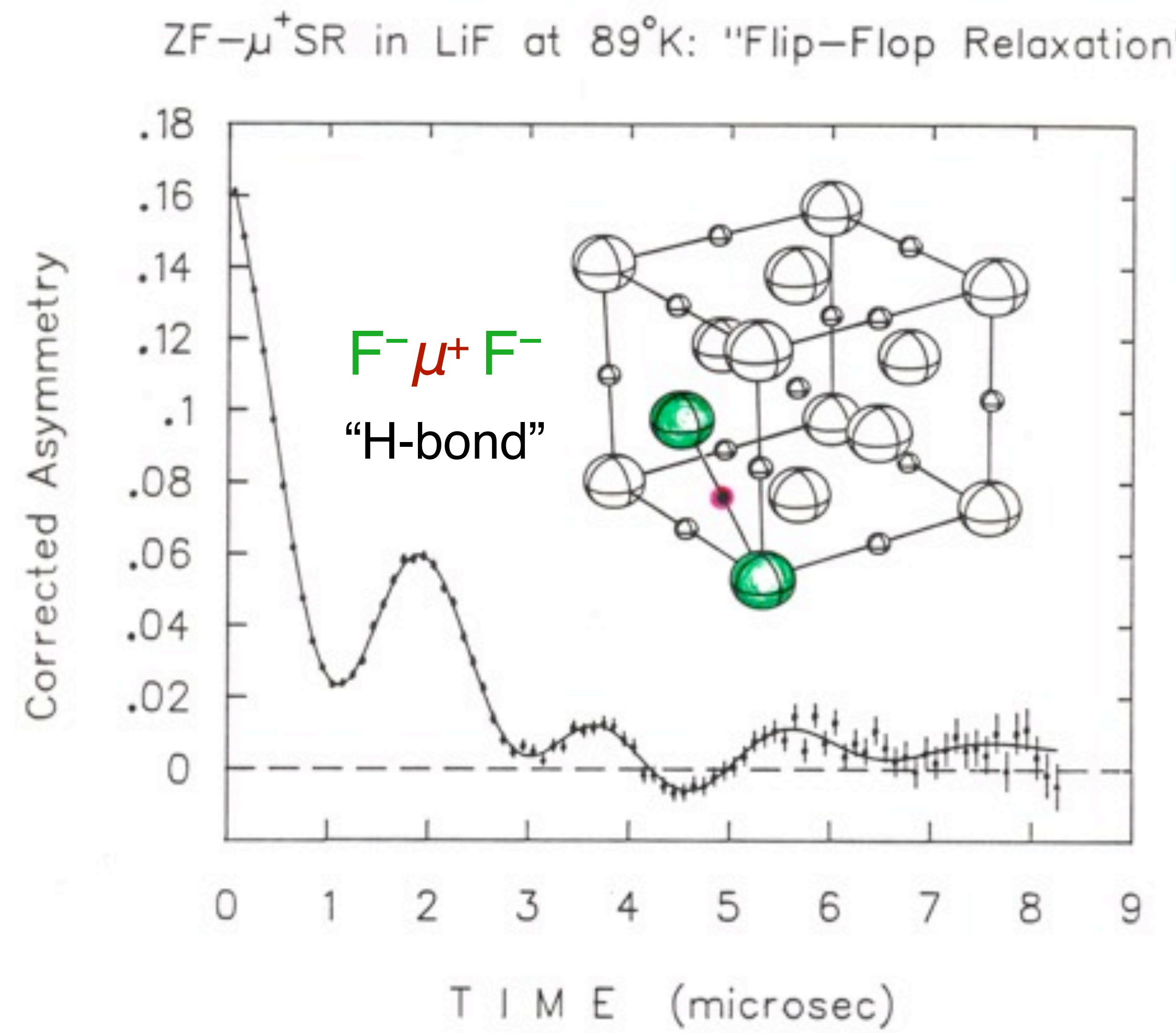
- 1983: Celio & Meier point out (a) actual lattice sum never gives true Gaussian or Lorentzian $\mathcal{D}(\vec{B}_{loc})$; (b) Host SPINS also "see" field from μ^+ moment
⇒ "STATIC" approximation is poor. Expect "FLIP-FLOP" OSCILLATIONS in $g_{zz}(t)$ -- small for MANY SPINS

Difficult to detect "Meier wiggles" in (e.g.) Cu -- 6 nn spin 3/2 ⇒ VERY MESSY SPIN HAMILTONIAN!

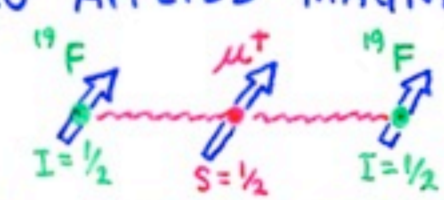


- 1984-5: "FμF" OSCILLATIONS found at TRIUMF. -- ideal, simplest case!

ZF–μ^+SR in LiF at 89°K: "Flip–Flop Relaxation"



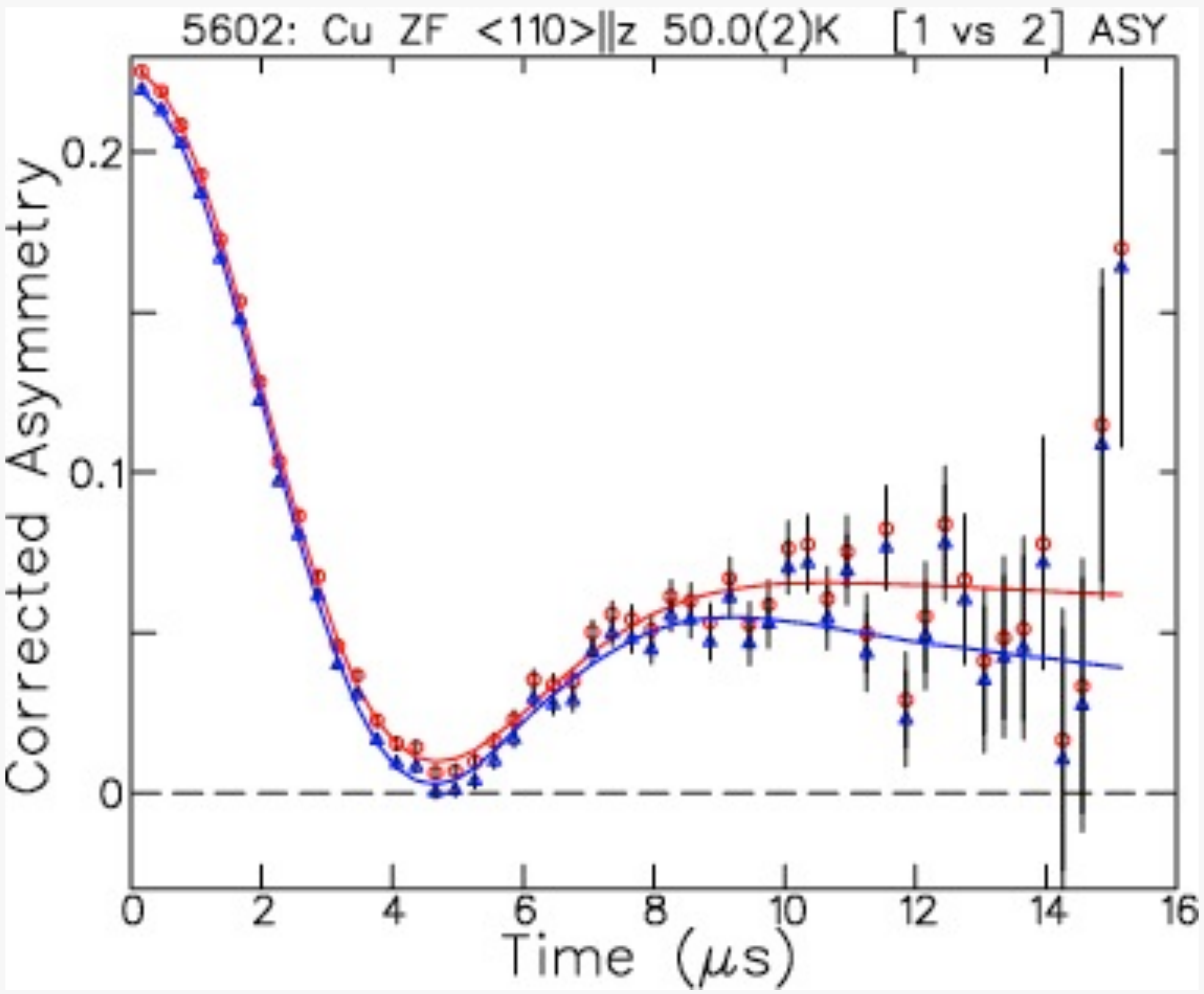
ZERO APPLIED MAGNETIC FIELD:

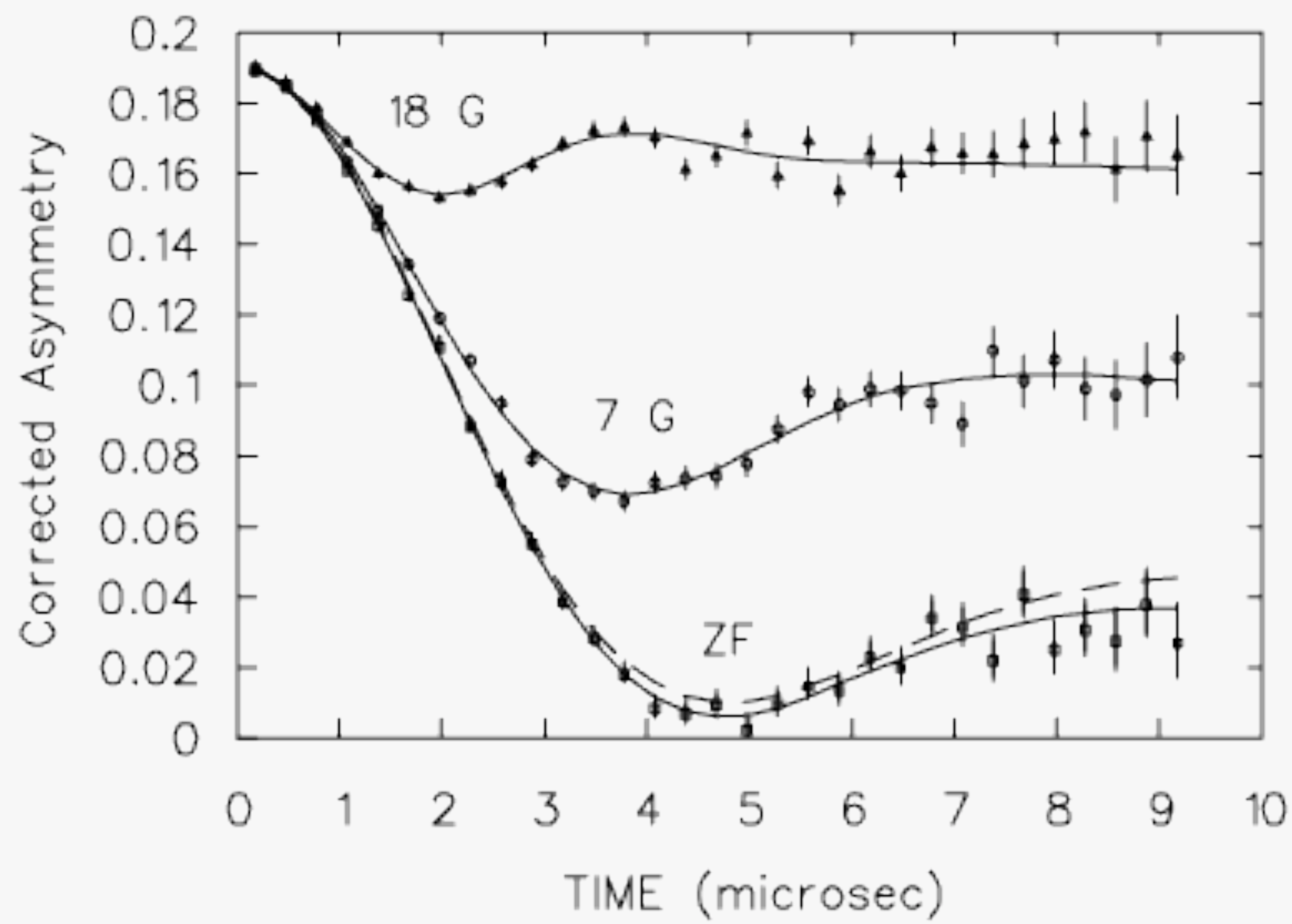


PURE MAGNETIC DIPOLE-DIPOLE INTERACTIONS LEAD TO COHERENT OSCILLATIONS.

$$G_{zz}(t) = \frac{1}{6} e^{-\Lambda(t)} \left\{ 3 + \cos \sqrt{3}\,\omega t + \left(1 - \frac{1}{\sqrt{3}}\right) \cos\left[\frac{3-\sqrt{3}}{2}\omega t\right] + \left(1 + \frac{1}{\sqrt{3}}\right) \cos\left[\frac{3+\sqrt{3}}{2}\omega t\right] \right\}$$

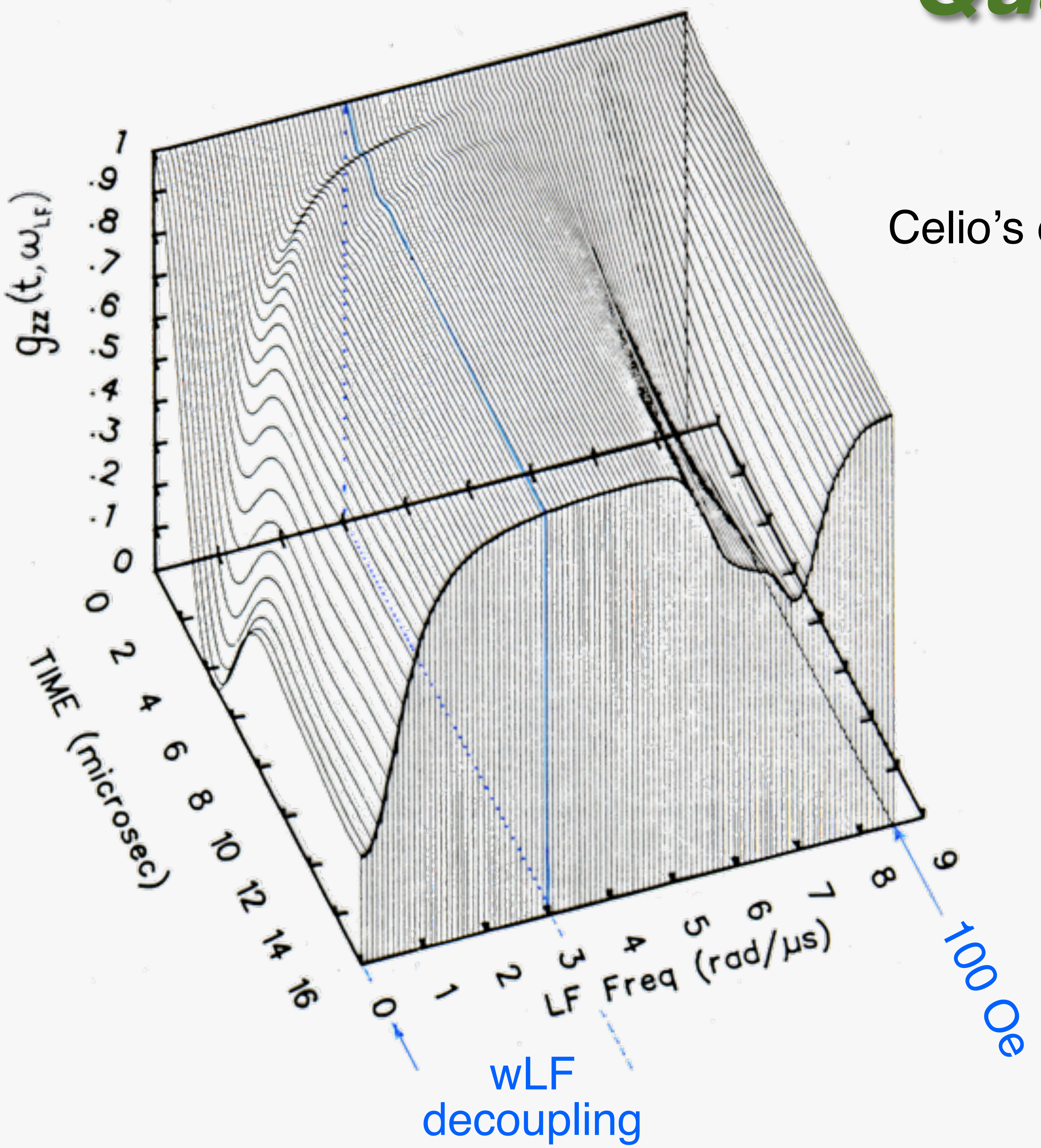
Dynamic Gaussian Kubo-Toyabe *vs.* ***actual $G_{zz}(t)$ in Cu***





Quadrupolar μALCR in Cu <100> || *B*

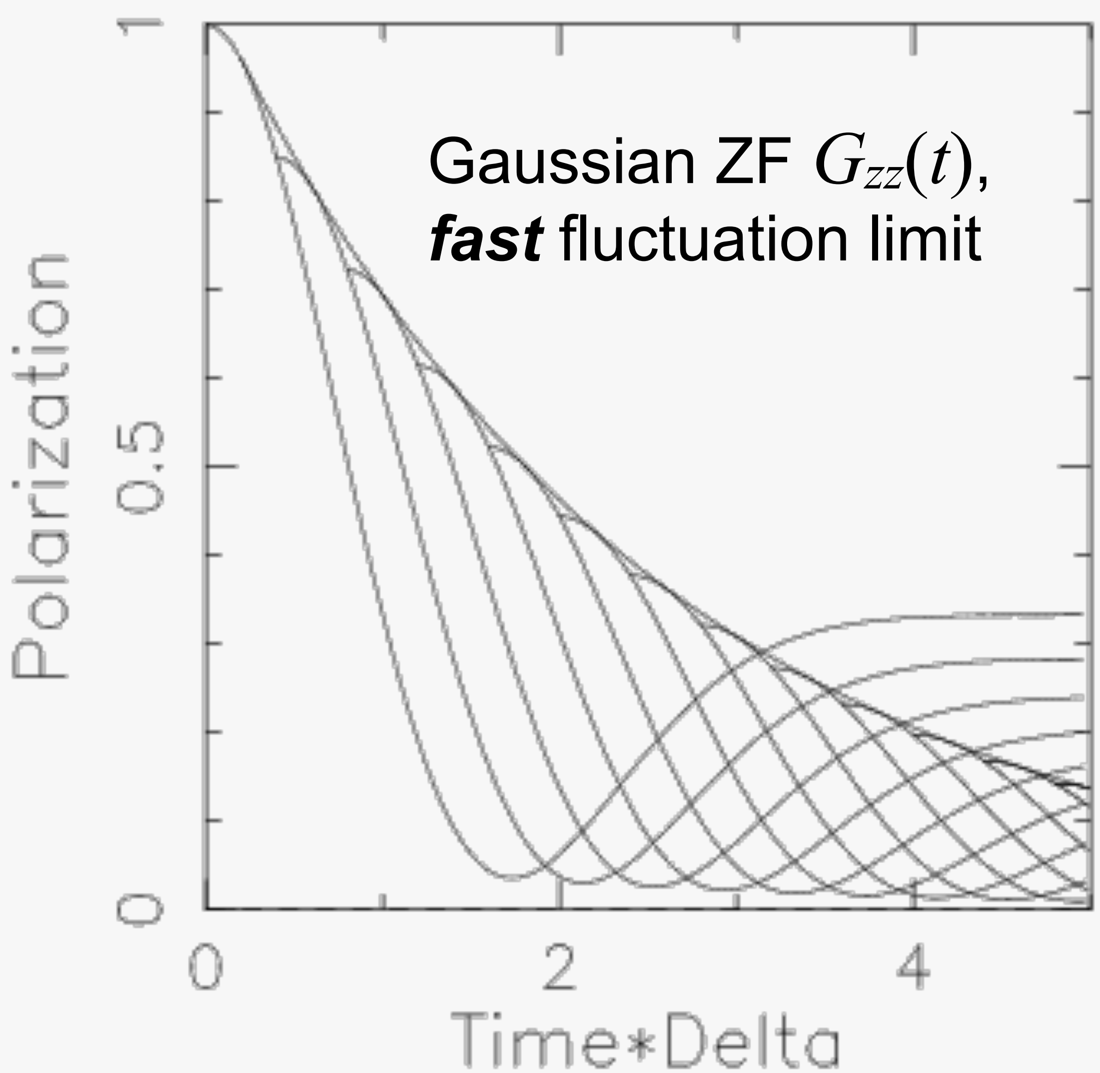
Celio's calculation of static "relaxation" function



Wednesday, August 10, 2011

Motion of μ⁺ Spins in Fluctuating Local Fields

"Strong Collision" model: local field is reselected at random from the same distribution each time a fluctuation takes place, either from muon hopping (plausible) or from reorientation of nearby moments (unlikely to change so completely).



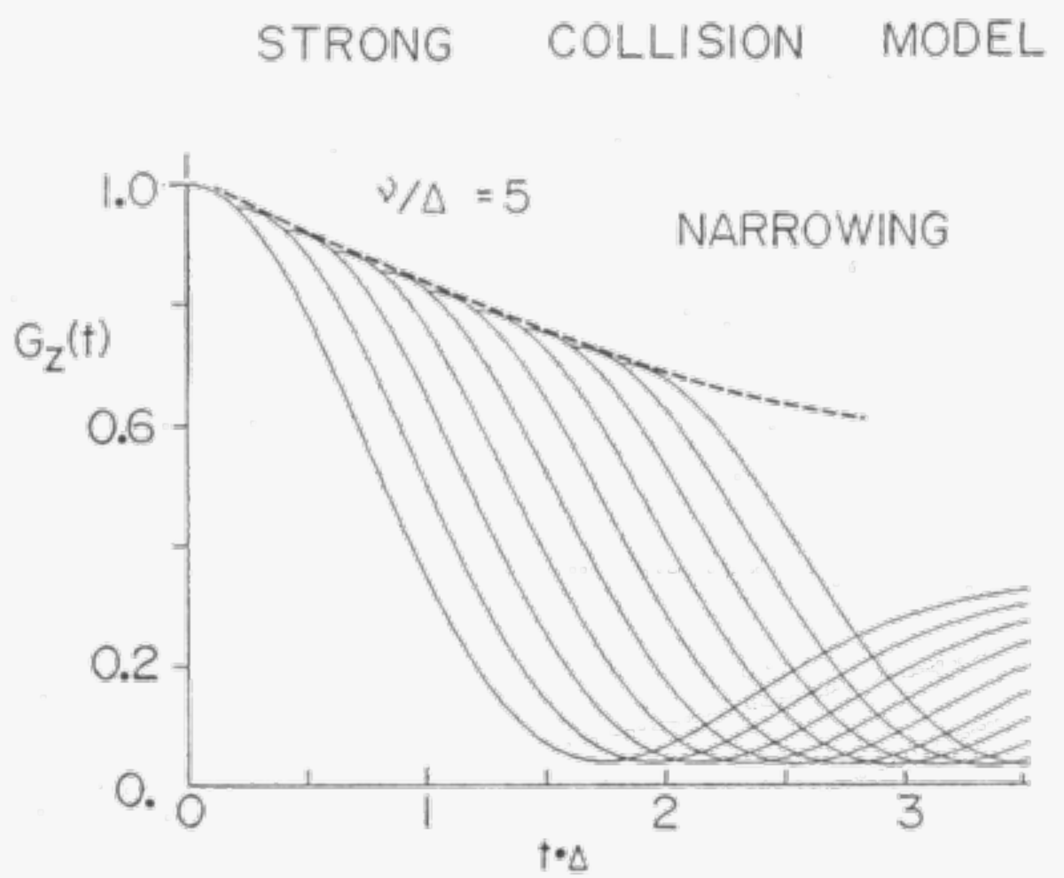
Kehr's recursion relation:

$$G(\Delta, t, \nu) = g(\Delta, t) e^{-\nu t} + \nu \int_0^t G(\Delta, t-\tau) g(\Delta, \tau)\, e^{-\nu\tau}\, d\tau$$

Sometimes solvable using Laplace transforms; numerical methods usually work too.

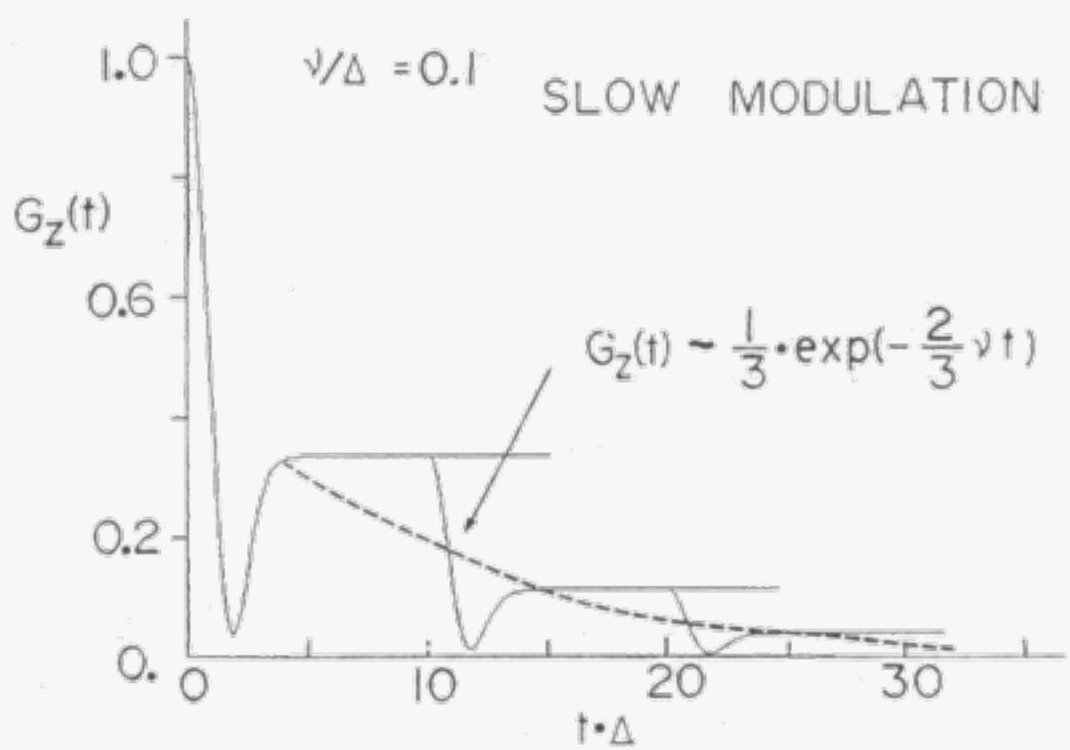
Used to extract "**hop**" or **fluctuation** rate ν.

Motion of μ^+ Spins in Fluctuating Local Fields



Gaussian ZF $G_{zz}(t)$,
fast fluctuation limit:
"motional narrowing".

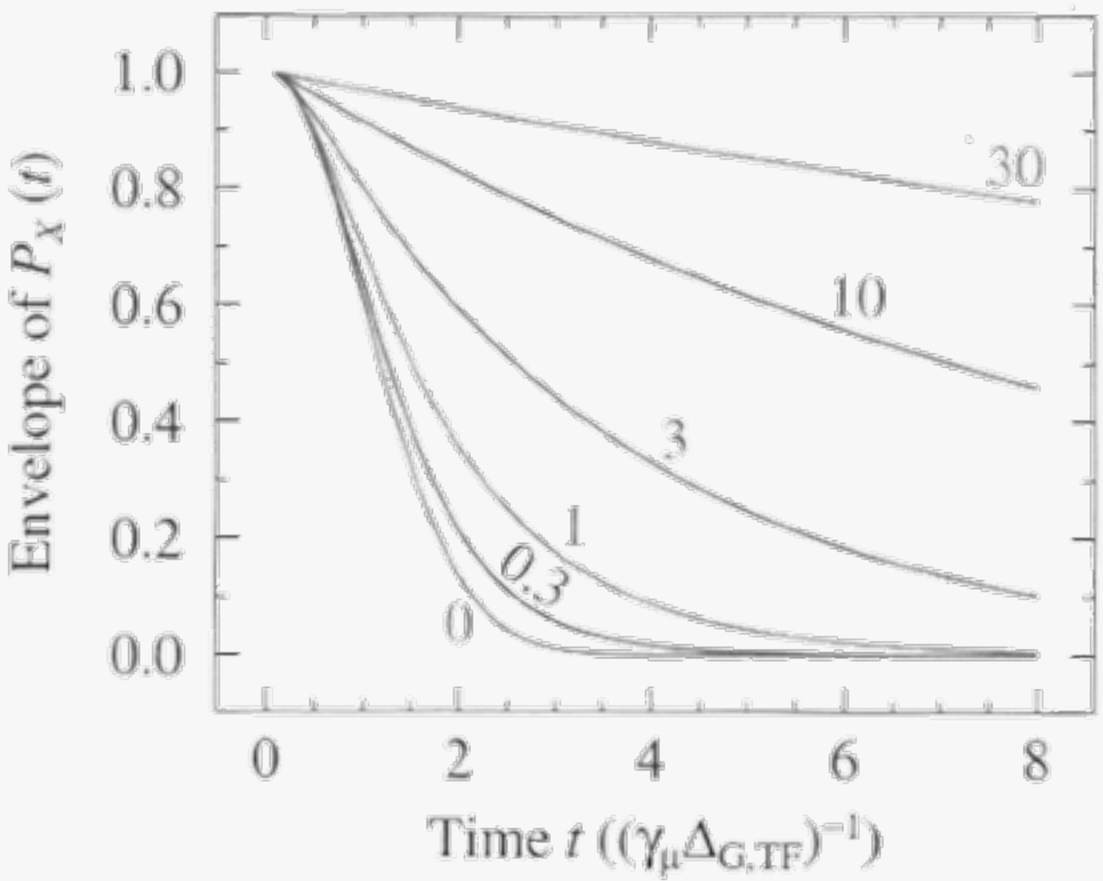
(Cartoons from Y.J. Uemura's PhD thesis, 1981)



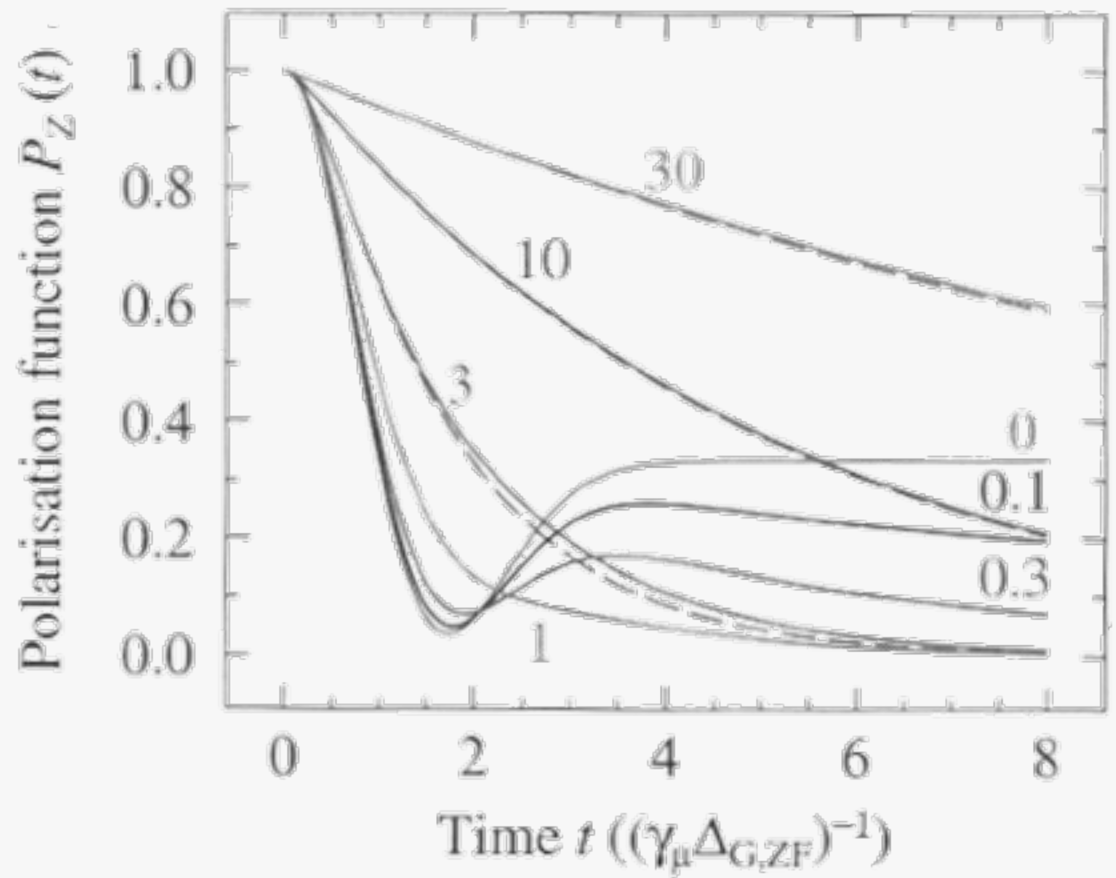
Gaussian ZF $G_{zz}(t)$,
slow fluctuation limit:
only the "1/3 tail" relaxes.

Relaxation in Fluctuating Fields

TF

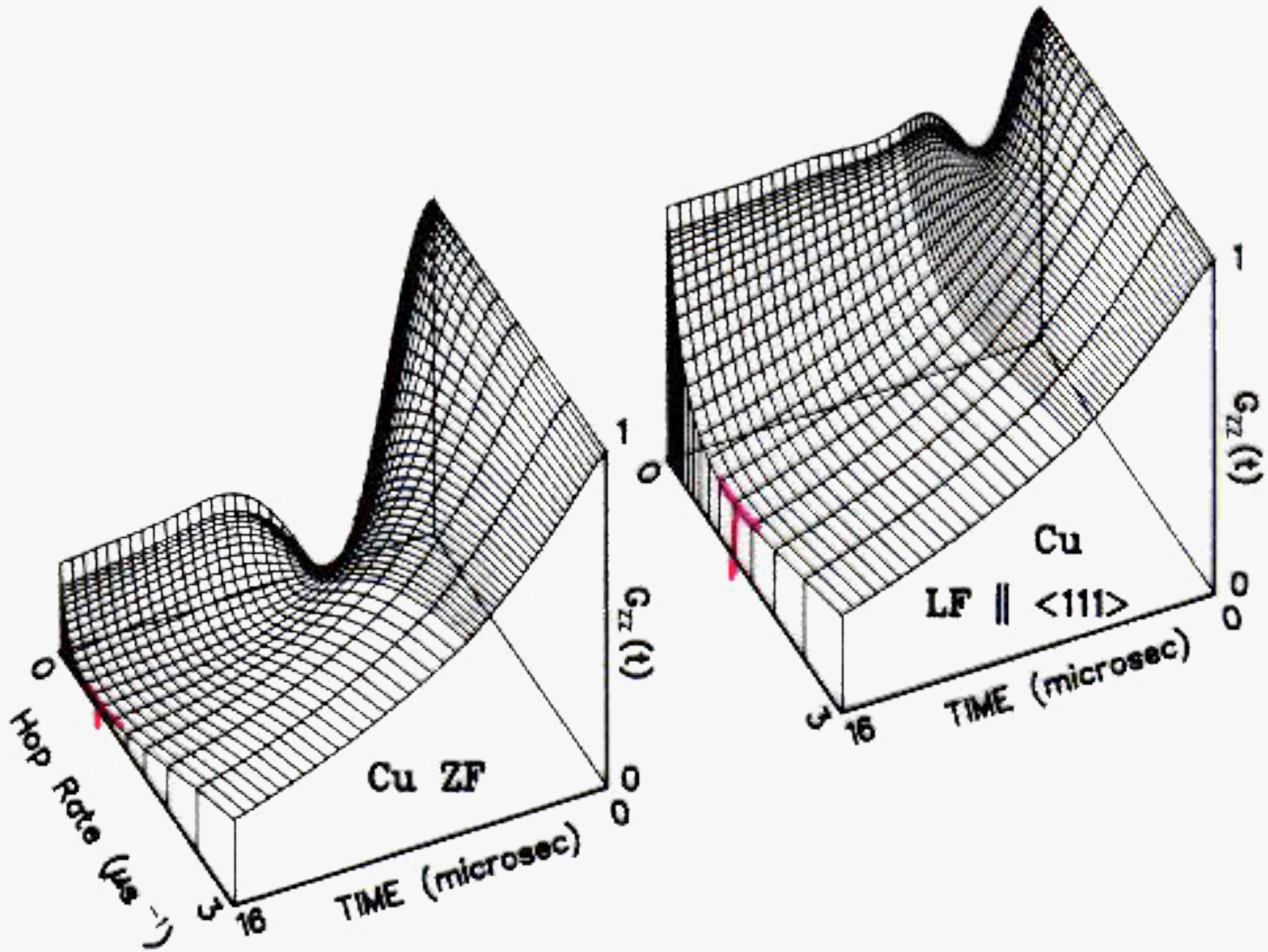


ZF

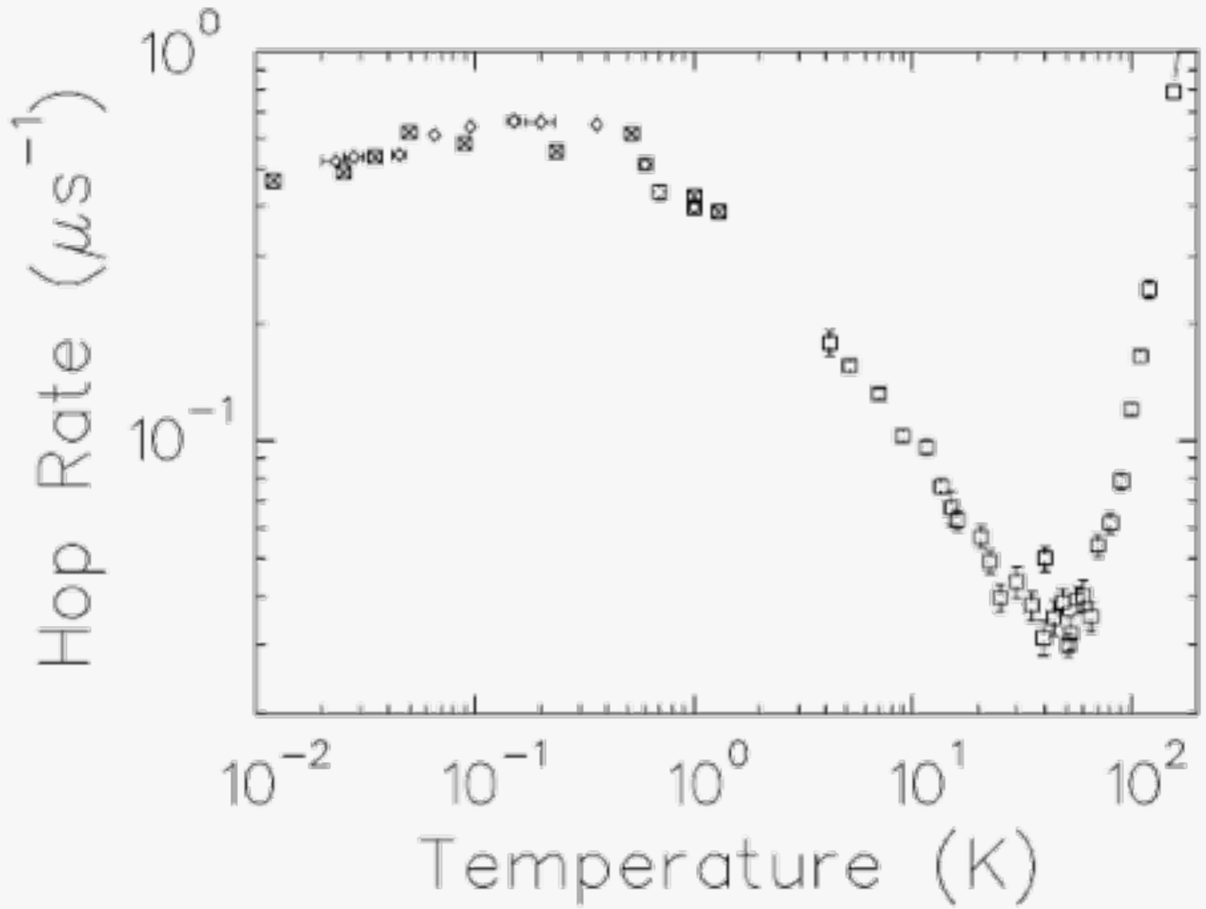


(From new book by Yaouanc & Dalmas de Reotier)

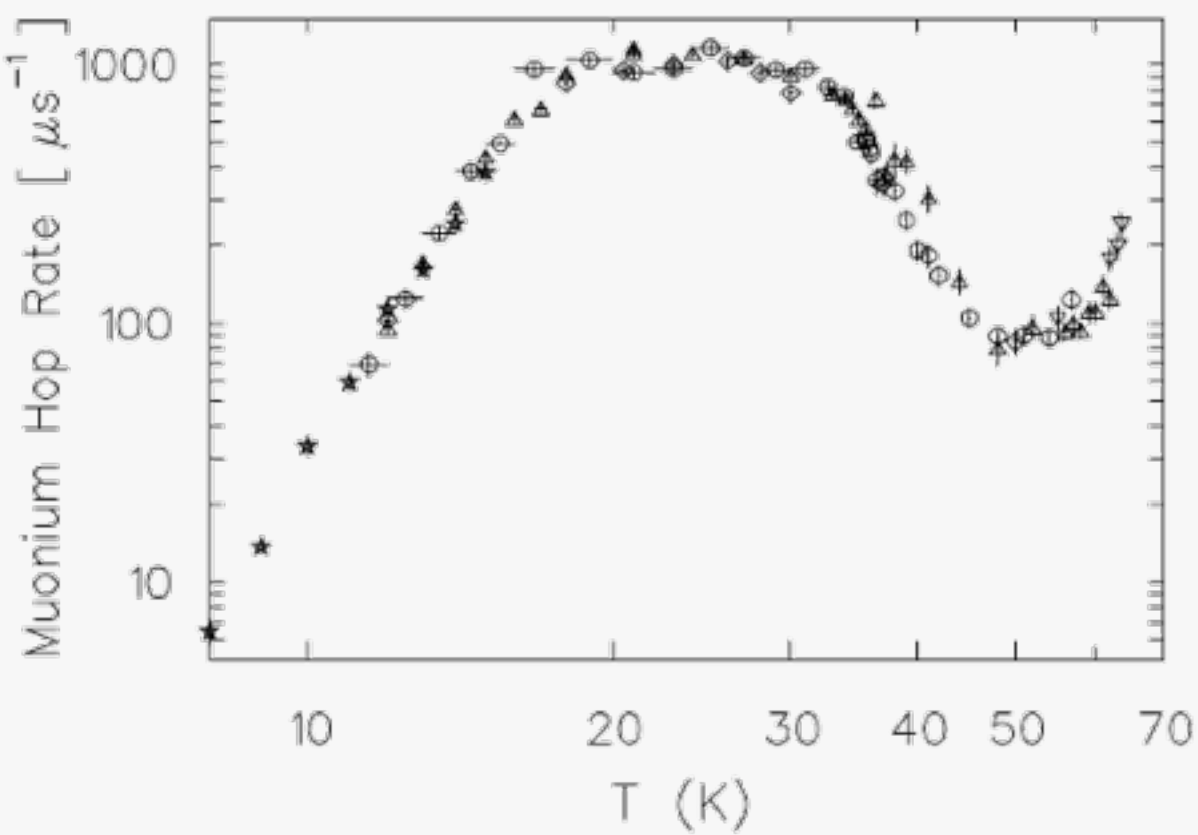
Dynamic Gaussian Kubo-Toyabe $G_{zz}(t)$ in Cu: ZF *vs.* LF



Universal Features in Quantum Diffusion

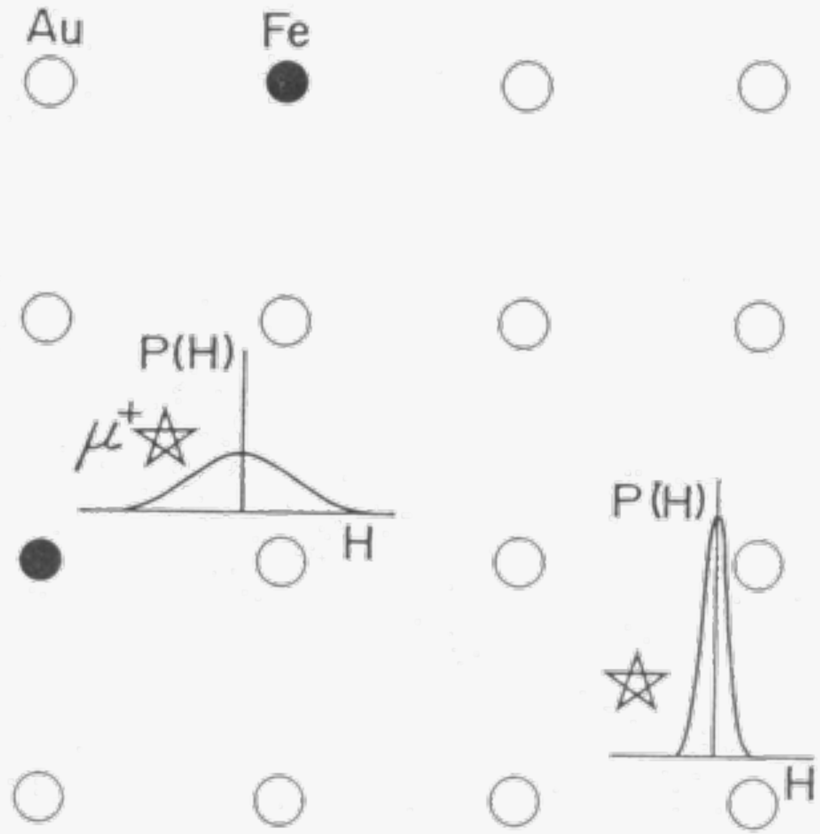


μ^+ hop rate
in “pure” Cu crystal



Muonium hop rate
in “pure” N_2 crystal

Dilute Spin Glass $G_{zz}(t)$:



(from Y.J. Uemura's PhD thesis, 1981)

Sub-ensembles of muons at similar distances from Fe spins see **Gaussian** distributions of *H* with different static widths Δ_G:

$$\mathcal{D}_G(H_i) = \frac{\gamma_\mu}{\Delta_G\sqrt{2\pi}} \exp\left(-\frac{\gamma_\mu^2 H_i^2}{2\Delta_G^2}\right), \quad i = \{x, y, z\}$$

$$\mathcal{D}_G(|\vec{H}|) = \left(\frac{\gamma_\mu}{\Delta_G\sqrt{2\pi}}\right)^3 \exp\left(-\frac{\gamma_\mu^2 H^2}{2\Delta_G^2}\right) \cdot 4\pi H^2$$

$$\Delta_z^2 = \frac{2}{3} I(I+1)\hbar^2\gamma_I^2\gamma_\mu^2 \sum_i r_i^{-6}$$

(5/2 times larger than Van Vleck limit in HTF)

Gaussian distribution of static Gaussian widths Δ_G *gives* an overall **Lorentzian**:

$$\mathcal{D}_{SG}(\Delta_G) = \sqrt{\frac{2}{\pi}} \cdot \frac{\Delta_L}{\Delta_G^2} \exp\left(-\frac{\Delta_L^2}{2\Delta_G^2}\right)$$

$$\mathcal{D}_L(H_i) = \int_0^\infty \mathcal{D}_G(H_i)\mathcal{D}_{SG}(\Delta_G)\, d\Delta_G \longleftrightarrow$$

But . . .
overall field distribution in static limit is **Lorentzian**:

$$\mathcal{D}_L(H_i) = \frac{\gamma_\mu}{\pi} \cdot \frac{\Delta_L}{\Delta_L^2 + \gamma_\mu^2 H_i^2}, \quad i = \{x, y, z\}$$

$$\mathcal{D}_L(|\vec{H}|) = \frac{\gamma_\mu^3}{\pi^2} \cdot \frac{\Delta_L}{\Delta_L^2 + \gamma_\mu^2 H_i^2} \cdot 4\pi H^2$$

Wednesday, August 10, 2011

TRIUMF

Spin Glass *Dynamics*:

For a given fluctuation rate ***ν***,

$$G_{zz}^{DSG}(t,\Delta_L,\nu)\int_0^\infty G_{zz}^{GZF}(t,\Delta_G,\nu)\mathcal{D}_{SG}(\Delta_G)\,d\Delta_G$$

But below the "glass temperature" T_g, local fields have a ***static*** part $\Delta_s=\sqrt{Q}\cdot\Delta_L$

and a ***dynamic*** part $\Delta_d=\sqrt{1-Q}\cdot\Delta_L$, where Q is a sort of "spin glass order parameter".

Since these two parts are statistically independent, they produce an overall relaxation function that is the **product** of the static and dynamic functions:

$$G_{zz}^{SG}(t,\Delta_L,\nu,Q)=g_{zz}^{L}(t,\Delta_s)\cdot G_{zz}^{DSG}(t,\Delta_d,\nu)$$

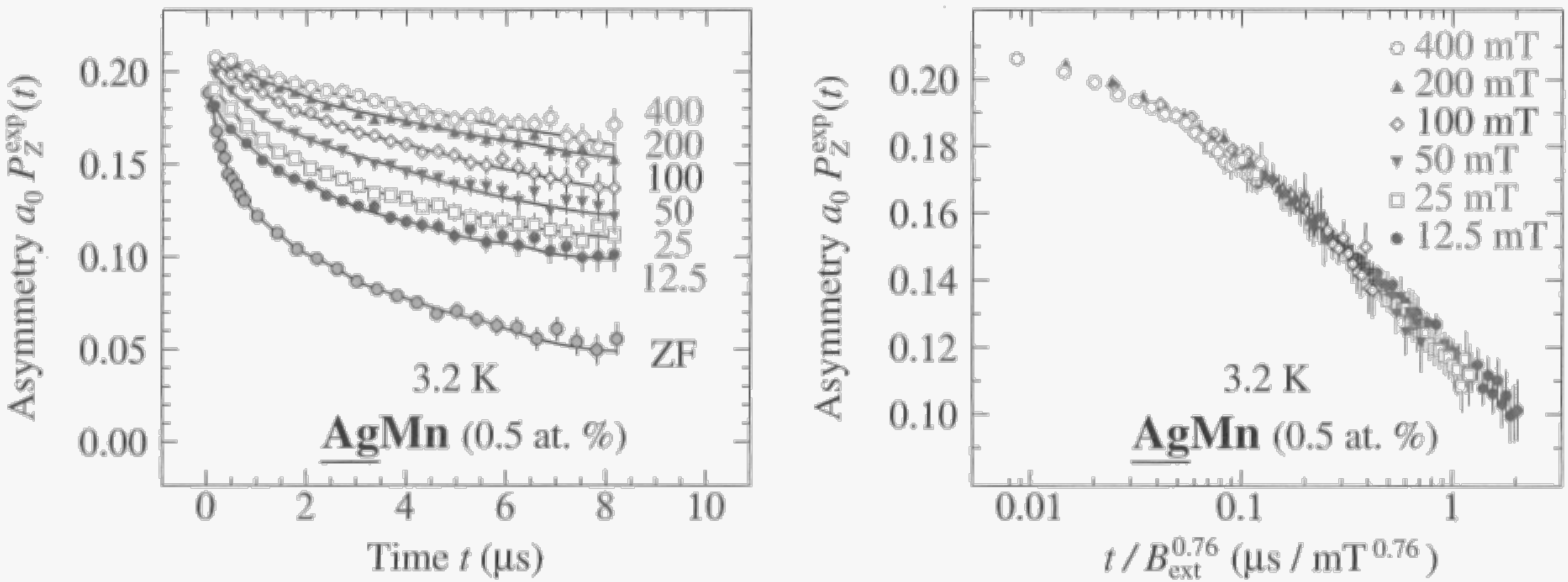
In his 1981 PhD thesis, Y.J. Uemura solved the above equations for the overall ZF SG $G_{zz}(t)$:

$$\begin{aligned}G_{zz}^{SG}(t,\Delta_L,\nu,Q) &= \frac{1}{3}\exp\left(-\sqrt{4\Delta_d^2\,t/\nu}\right)\\ &+ \frac{2}{3}\left(1-\frac{\Delta_s^2\,t^2}{\sqrt{4\Delta_d^2\,t/\nu+\Delta_s^2\,t^2}}\right.\cdot\\ &\qquad\left.\exp-\sqrt{4\Delta_d^2\,t/\nu+\Delta_s^2\,t^2}\right)\end{aligned}$$

Note the famous "**root exponential**" time dependence as $Q\rightarrow 0$ (*i.e.* above T_g). Below T_g the behavior of $G_{zz}(t)$ is "volatile" . . . .

"*Stretched Exponentials*"

Legitimate Uses **: Spin Glass Dynamics**



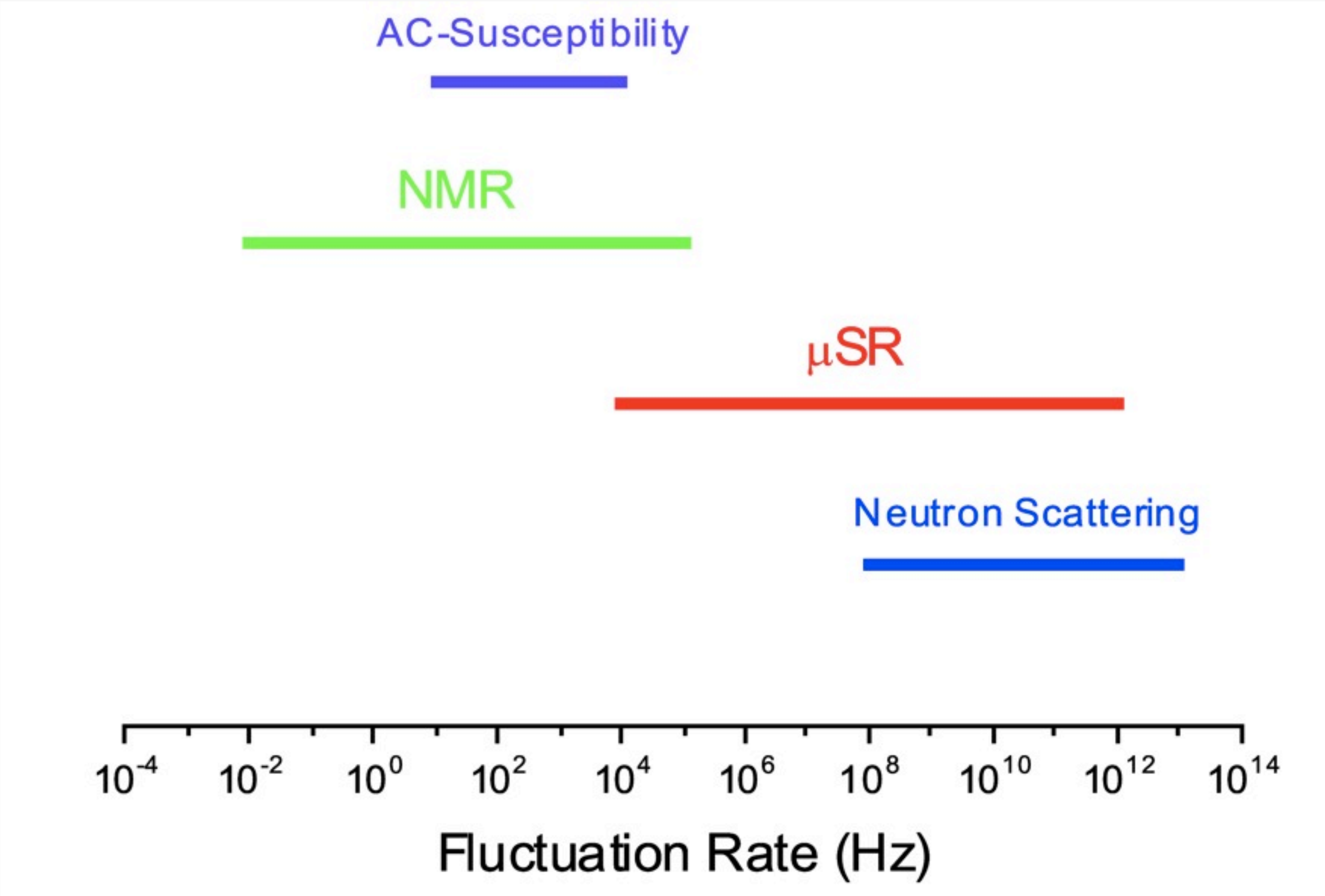
Bogus Empiricism **: Everything Else**

(*It will fit anything, but it means nothing!*)

Generalized static ZF ***g****$_{zz}$*(*t*) :

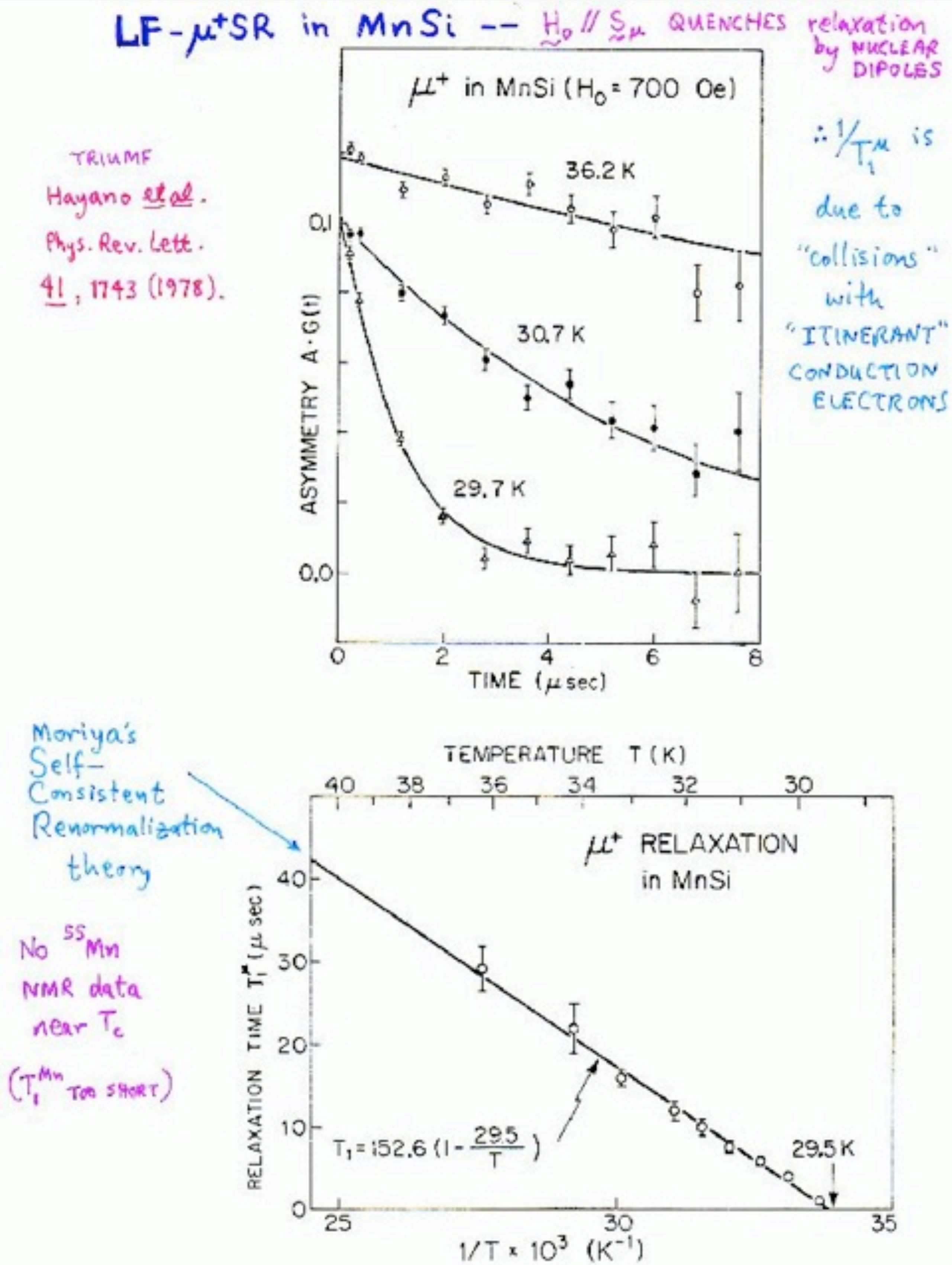
$$g_{\mathrm{gen}}^{ZF}(\Delta, t) = \frac{1}{3} + \frac{2}{3} \cdot \left[1 - (\Delta\, t)^{\beta}\right] \cdot \exp\left[-\frac{(\Delta\, t)^{\beta}}{\beta}\right]$$

Time Scales



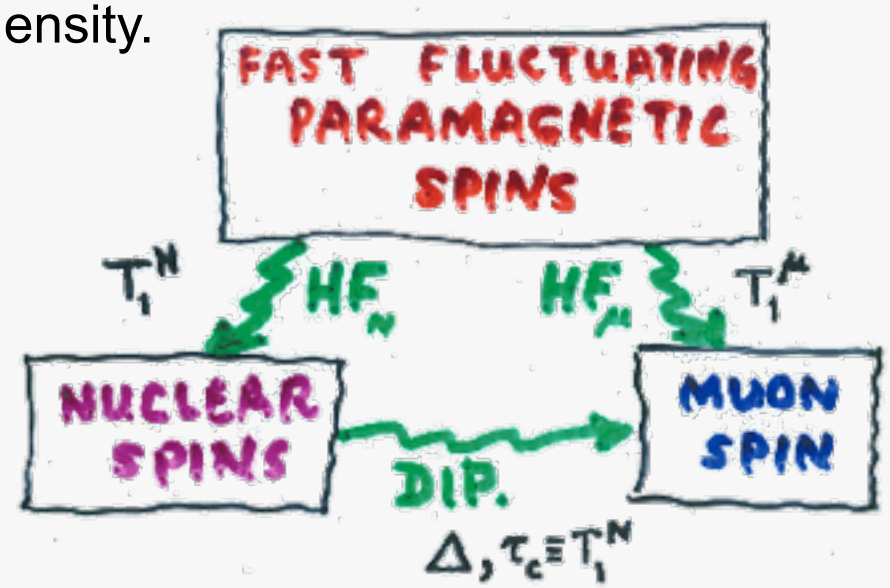
AC-Susceptibility
NMR
μSR
Neutron Scattering
10^{-4} 10^{-2} 10^{0} 10^{2} 10^{4} 10^{6} 10^{8} 10^{10} 10^{12} 10^{14}
Fluctuation Rate (Hz)

“Amazing Manganese Silicide”



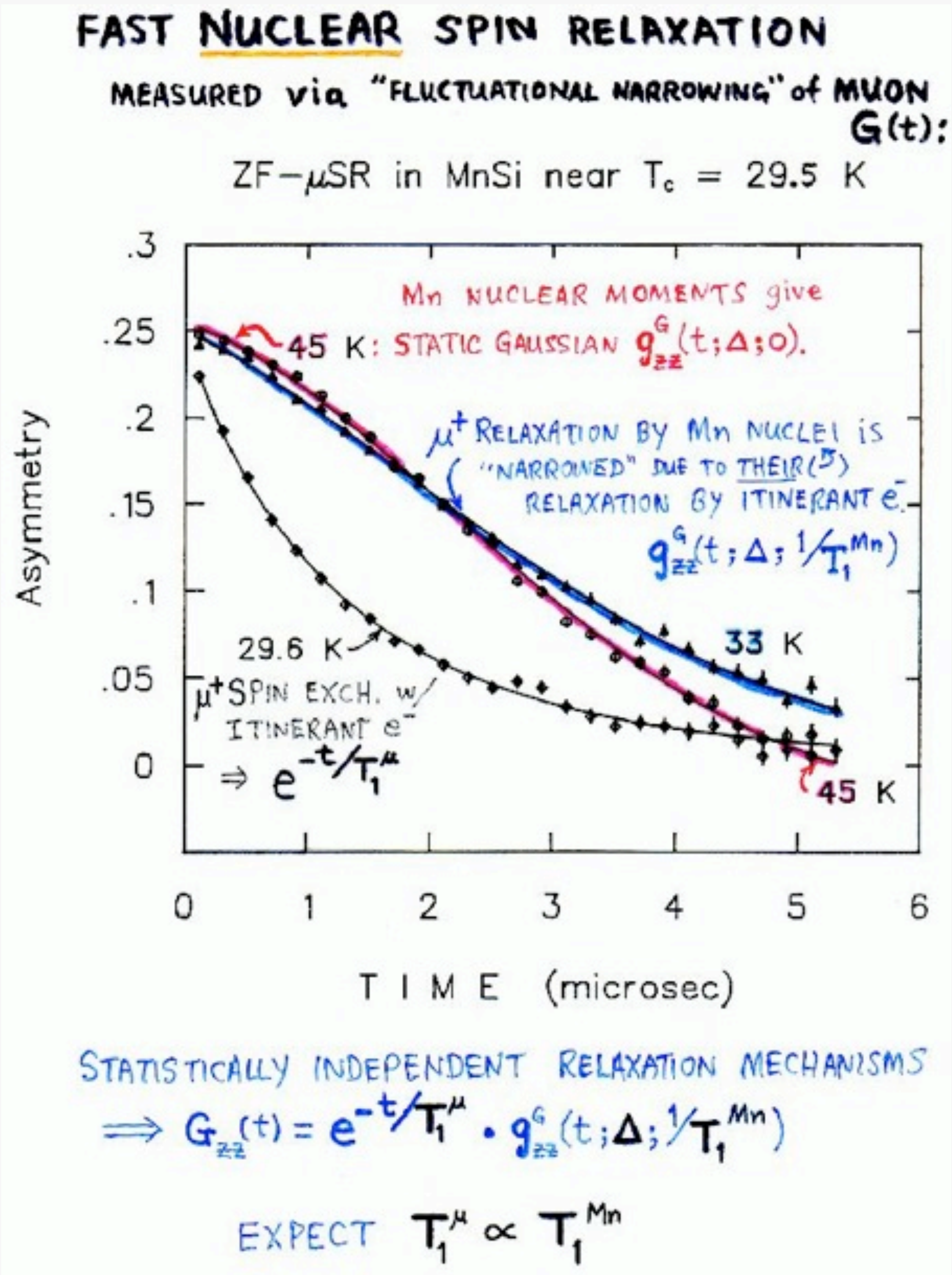
MnSi is emphatically *magnetic*; nevertheless, it was the first substance in which the static gaussian Kubo-Toyabe relaxation function was ever observed. This is because the paramagnetic spin density fluctuates so fast at room temperature that it is effectively decoupled from the muon and the ^{55}Mn nuclei, leaving the latter to generate random static dipolar fields that produce a gaussian distribution at the muon in ZF.

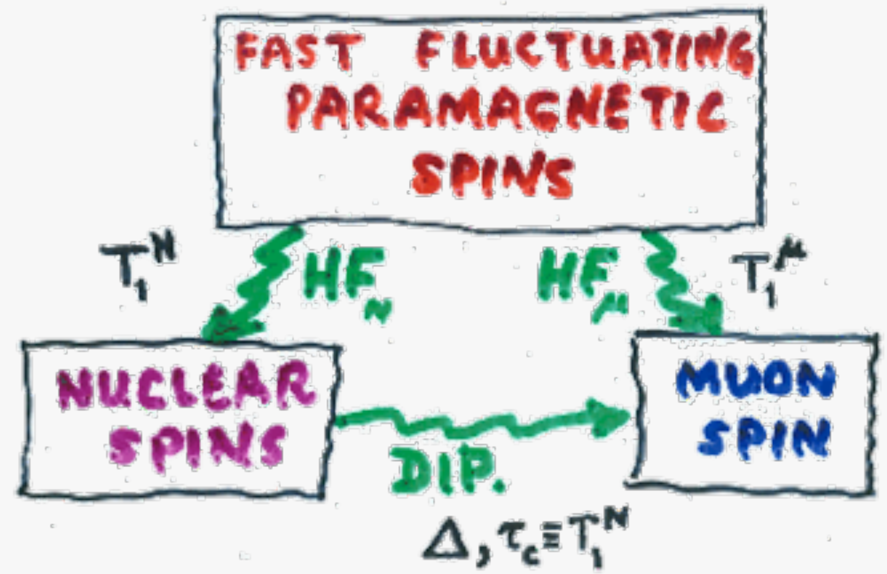
At lower *T* the muon is strongly relaxed by the *large* fluctuating fields of the paramagnetic conduction electron spin density.



Where ^{55}Mn NMR had failed to get within 100K of the critical temperature, *μSR* was able to follow the critical slowing down of spin density fluctuations down to within half a degree of T_c and confirm Moriya's self-consistent renormalization (SCR) theory.

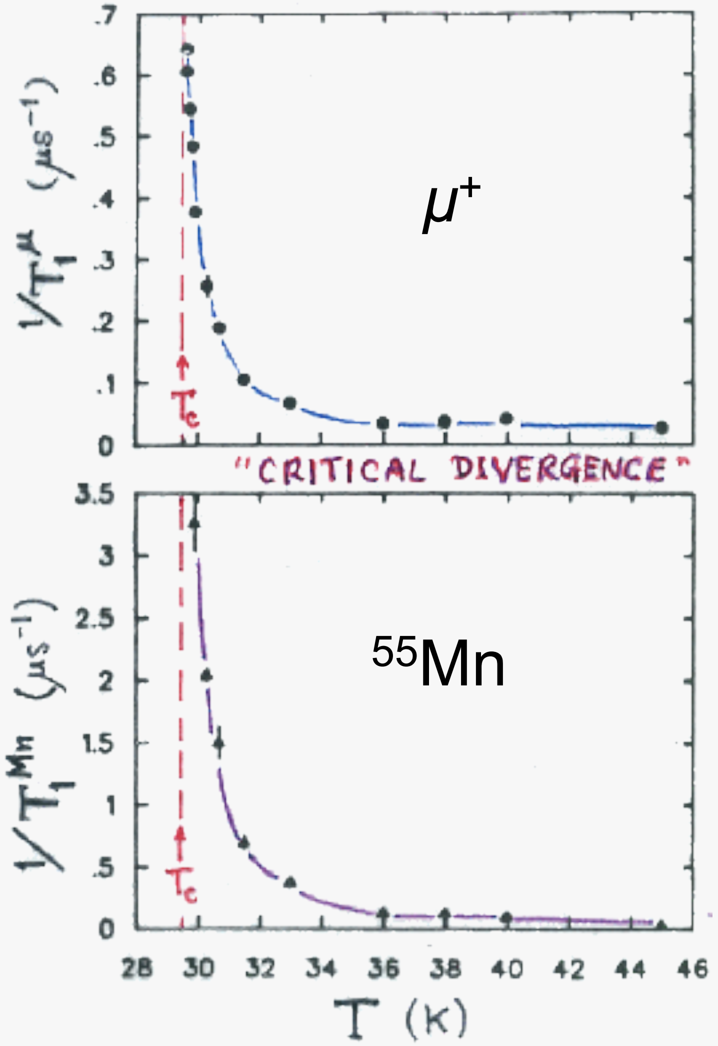
“Double Relaxation”





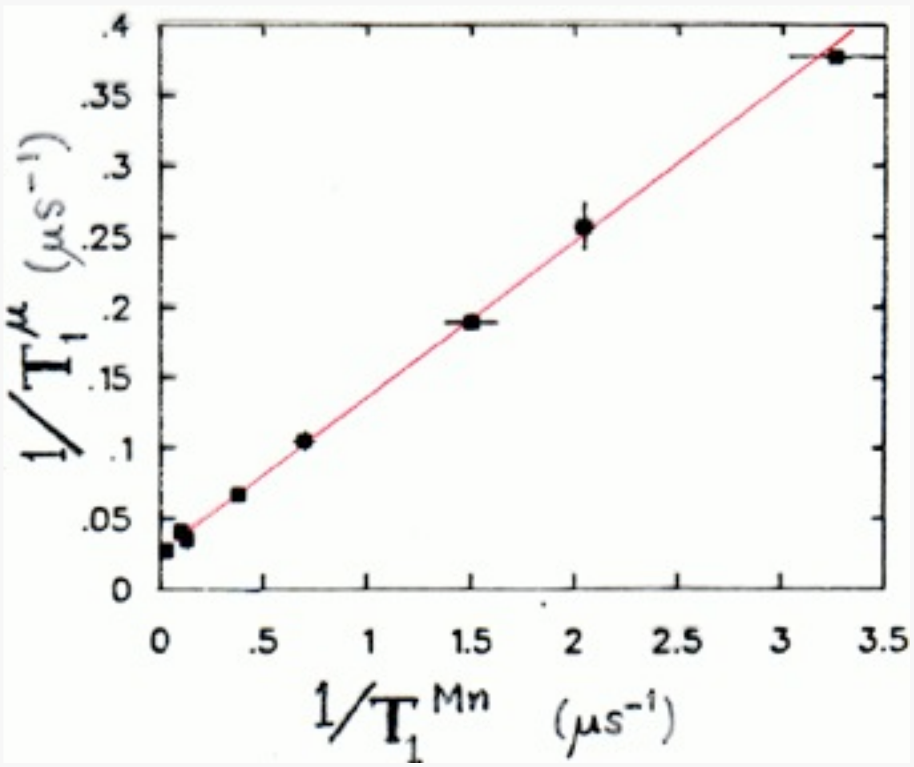
Now consider what happens when the paramagnetic spin density fluctuates at an ***intermediate*** rate, causing the ^{55}Mn nuclear moments to fluctuate (relax) faster, while at the same time beginning to have a direct effect on the *muon* spins. . . . The unmistakeable characteristic signature of “**double relaxation**” of this type is when the muon polarization function falls **below** the (high temperature) static gaussian ZF $g_{zz}(t)$ at **early** times but rises **above** same at **late** times. This clearly *cannot* be due to a simple multiplication of $G_{zz}(t)$ by an additional exponential relaxation function representing the direct effect of the paramagnetic moments. Excellent fits are possible over the entire temperature range in ZF, using a constant value for Δ_G (due to Mn nuclear moments) and allowing the ^{55}Mn and μ^+ relaxation rates to vary independently.

“Double Relaxation”



Knowing that the muon relaxation at *high T* is dominated by nearly-static ^{55}Mn nuclear dipoles whereas that at *low T* is dominated by fluctuating paramagnetic moments, and in between *both* must be taken into account, a much better determination of the critical divergence of $T_1^{-1}(\mu)$ was possible.

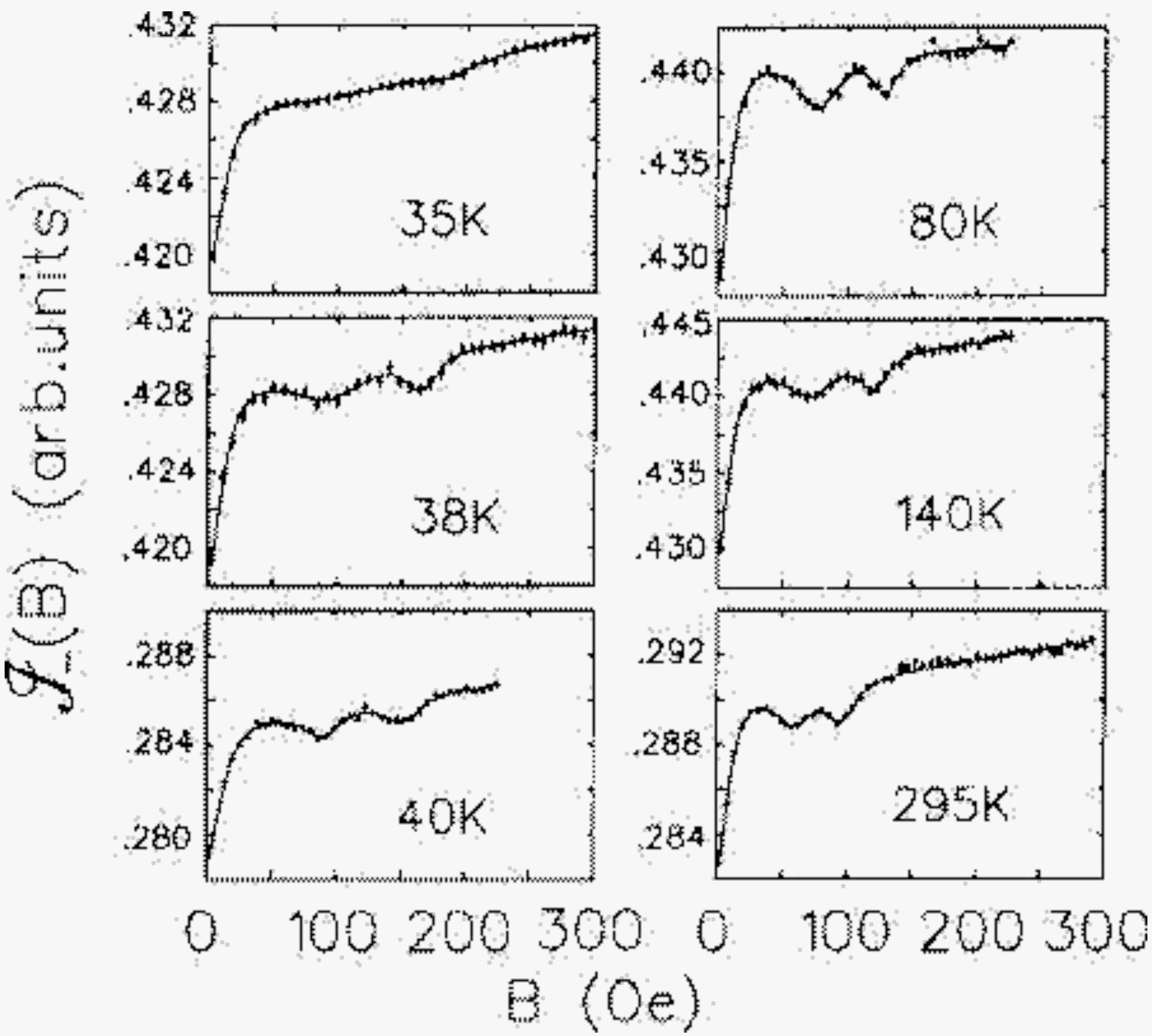
At the same time, by equating the fluctuation rate of the nuclear dipolar fields in $G_{KT}(t)$ with the relaxation rate of the ^{55}Mn nuclei, it was possible to obtain the critical divergence of T_1^{-1}(Mn) more than an order of magnitude closer to T_c than was possible using NMR.

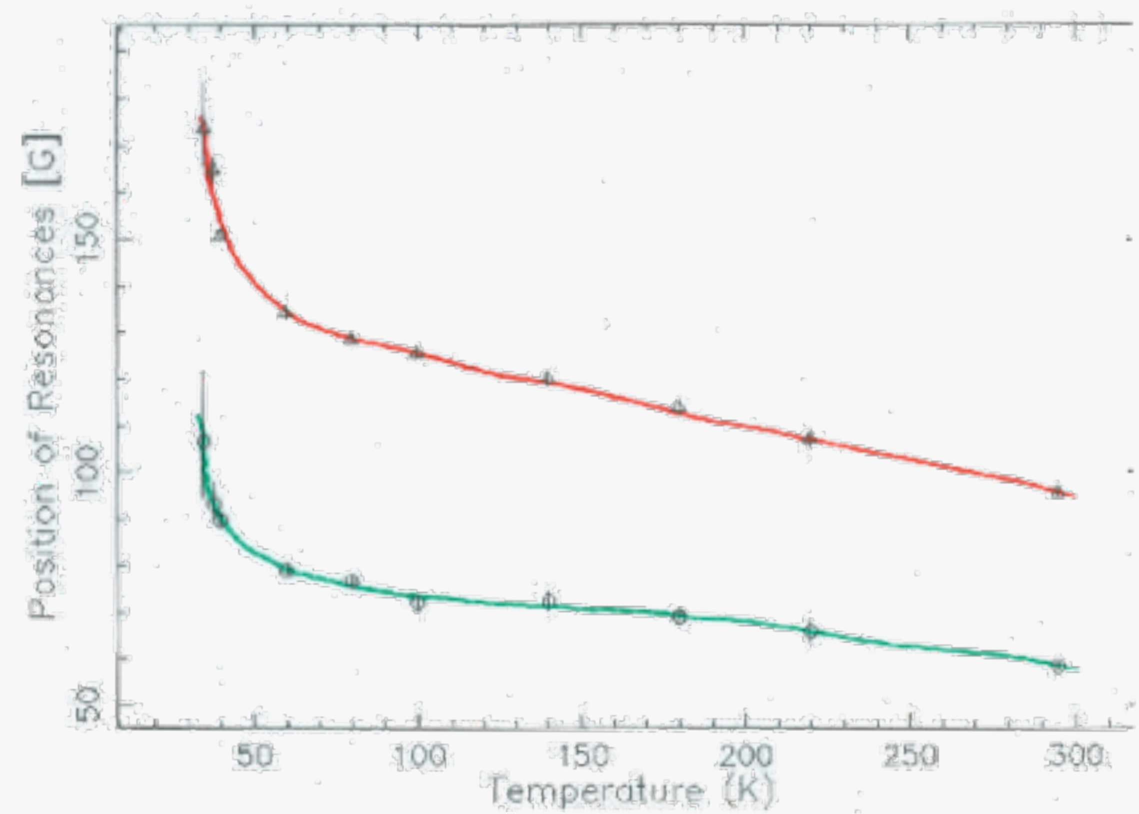


However, there is one further effect to account for. In order to obtain more reliable measurements of $T_1^{-1}(\mu)$ at the higher temperatures, a **LF** was applied to effectively *quench* the effect of the ^{55}Mn nuclear moments. Unfortunately, at certain fields this introduces yet *another* relaxation mechanism . . . .

Quadrupolar μALCR in MnSi

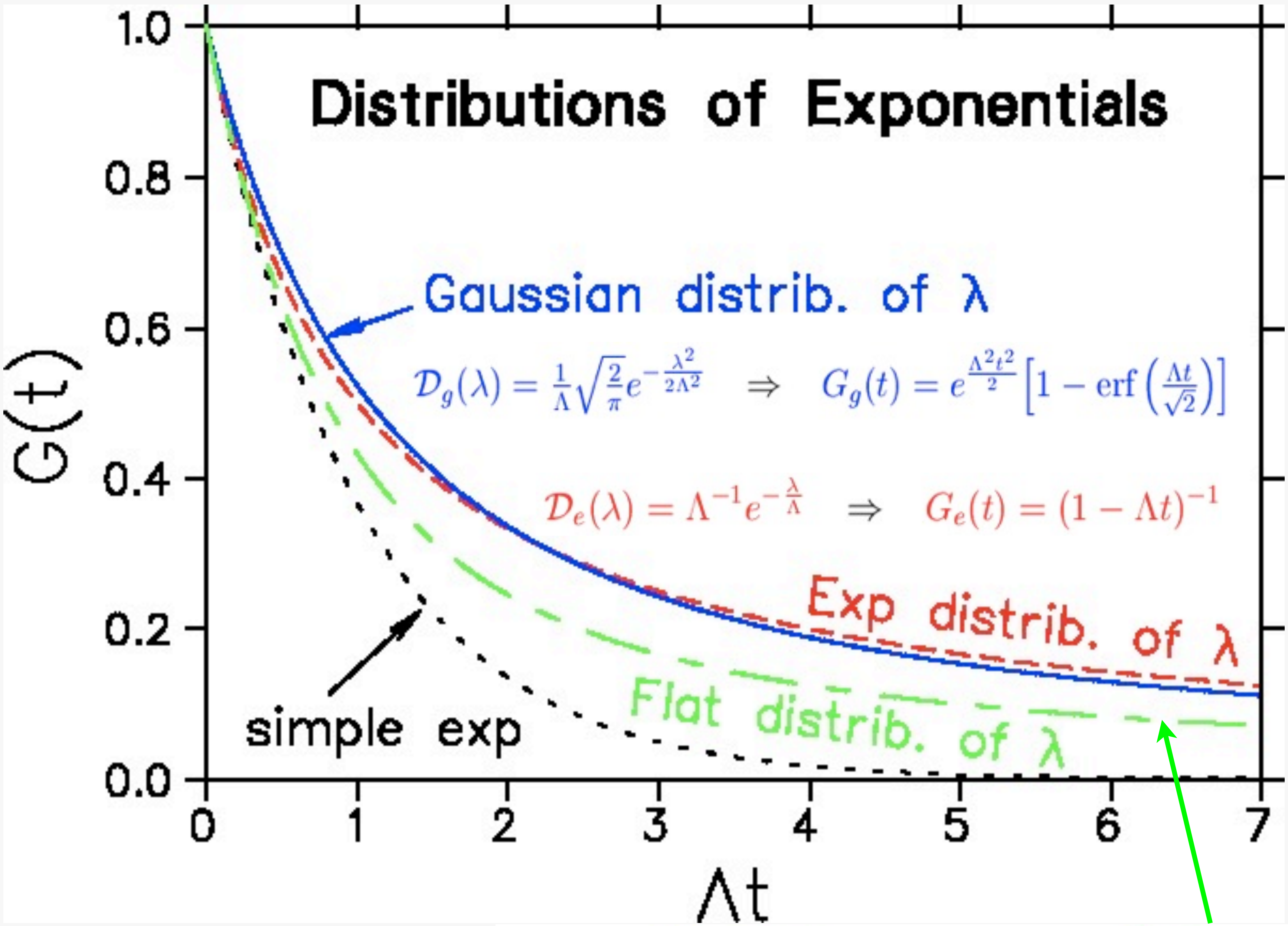
In manganese silicide at any temperature above T_c = 29.5K, the muon Zeeman splitting matches the Mn nuclear quadrupole splitting at two longitudinal magnetic fields (since ^{55}Mn is spin 5/2). The values of these fields are, perhaps surprisingly, strongly *temperature dependent*. This is attributed [R. Kadono *et al*., Phys. Rev. B **48**, 16803 (1993)] to critical behaviour of the electric field gradient in MnSi, due to the coupling between charge density and spin density fluctuations in this itinerant electron helimagnet.





See Moriya's self-consistent renormalization (SCR) theory [T. Moriya and A. Kawabata, J. Phys. Soc. Jpn. **34**, 639 (1973); **35**, 669 (1973); or T. Moriya, *Spin Fluctuations in Itinerant Electron Magnetism* (Springer, New York, 1985)].

"Exotic Forms"



$$\mathcal{D}_f(\lambda) = \begin{cases} \frac{1}{2\Lambda}, & 0<\lambda<2\Lambda \\ 0 & \text{otherwise} \end{cases} \Rightarrow G_f(t) = \frac{1}{2\Lambda t}\left[1 - e^{-2\Lambda t}\right]$$

Exotic Forms: Arrival-Time Distributions

Suppose the muons are initially in an environment where their polarization evolves in one way, but at random times (usually governed by Markovian processes and therefore exponentially distributed in time) they enter a second environment with a different polarization time dependence.

A typical example would be when the muon is initially in a muonium atom that later reacts chemically to form a diamagnetic compound. In this case the diamagnetic signal amplitude depends on whether the Mu react faster than its precession period, which is 1/103 of that of the free muon (and the precession is in the opposite direction). The resultant diamagnetic amplitude is called the **residual polarization**.

Another example would be where the muon diffuses through a crystal to a defect where the local magnetic field is significantly different from that in the pure host. This gives a similar result except that the criterion is that the average trapping time should be short compared with the *difference* between the two frequencies. In some cases **both** frequencies can be observed in the same spectrum.

Both (and many others) can be treated with Laplace transforms . . . but this can get complicated, and I have run out of time by now. :-)

Exotic Forms: **More Bogus Empiricism**

Oh, never mind.

There is more than enough bogus empiricism already.

C'est fini!

Books:

1. A. Yaouanc and P. Dalmas De Réotier, "*Muon Spin Rotation, Relaxation, and Resonance*", (Oxford University Press 2011).

Review articles:

1. J. Chappert, "*Principles of the μSR technique*", in *Muons and Pions in Materials Research*, ed. J. Chappert and R.I. Grynszpan (Elsevier 1984).
2. S.F.J. Cox, "*Implanted muon studies in condensed matter science*", J. Phys. C 20 (1987) 3187.
3. A. Schenck and F.N. Gygax, "*Magnetic materials studied by muon spin rotation spectroscopy*", in *Handbook of Magnetic Materials, Vol. 9*, ed. K.H.J. Buschow (Elsevier 1995).
4. P. Dalmas de RJotier and A. Yaouanc, "*Muon spin rotation and relaxation in magnetic materials*", J. Phys. Cond. Matter 9 (1997) 9113.
5. A. Amato, "*Heavy fermion systems studied by μSR technique*", Rev. Mod. Phys. 69 (1997) 1119.
6. R.H. Heffner, "*Muon spin relaxation studies of small-moment heavy-fermion systems*", in *Magnetism in Heavy Fermion Systems*, ed. H.B. Radousky (World Scientific 2000).
7. G.M. Kalvius, D.R. Noakes and O. Hartmann, "*μSR studies of rare earth and actinide magnetic materials*", *Handbook on the Physics and Chemistry of Rare Earths*, Vol. 32 (Elsevier, 2001).

Research articles on muon spin relaxation functions:

• *Uemura and collaborators discover Gaussian KT relaxation in MnSi*: Phys. Rev. Lett. **41** (1978) 1743.

• *Uemura dilute-alloy spin-glass dynamics*: Phys. Rev. B **31** (1985) 546.

• *How does the Gaussian distribution for dense-moment systems evolve toward the Lorentzian as moment-concentration is reduced*? Phys. Rev. B **44** (1991) 5064.

• Strange case #1: *Observed: static Lorentzian KT relaxation in well-ordered dense-moment cubic antiferromagnets*: [Uppsala] Hyperfine Int. **31** (1986) 303, [Oxford] Hyperfine Int. **64** (1990) 453, J. Magn. & Magn. Mater. **236** (2001) 198.

• Strange case #2: *Observed: A fast-fluctuation relaxation function with Gaussian shape*. Phys. Rev. Lett. **73** (1994) 3306.

• Strange case #3: *Observed: Monotonic relaxation to the 1/3 tail (static fields)*. Phys. Lett. A 199 (1995) **107**, Phys. Rev. B 56 (1997) 2352, J. Phys. Cond. Matter **11** (1999) 1589.

• Strange case #4: *Observed: "Power-exponential" relaxation in dense-moment spin glasses*. [Uppsala] Hyperfine Int. **31** (1986) 303, Phys. Rev. Lett. **72** (1994) 1291, Phys. Rev. B **65** (2002) 13-2413.

• Strange case #5: *Observed: true strong-collision-dynamic Lorentzian KT relaxation*. [Les Diablerets] Physica B **289-290** (2000) 303.